國立中央大學八十七學年度碩士班研究生入學試題卷

所別: 數學研究所 不分組 科目: 機率 共一頁 第一頁

10% 1. For any integer $n \geq 2$. Show that

$$2 \cdot 1 \cdot \binom{n}{2} + 3 \cdot 2 \cdot \binom{n}{3} + \cdots + n \cdot (n-1) \binom{n}{n} = n \cdot (n-1)\, 2^{n-2}.$$

10% 2. A hat contains n coins, f of which are fair, and b of which are biased to land with heads with probability $\frac{2}{3}$, with $f + b = n$. A coin is drawn at random from the hat and tossed once. It lands heads. What is the probability that it is a biased coin?

20% 3. Let X and Y be independent random variables, each uniformly distributed on $(0,1)$. Calculate

(a) $P\left(\left| \frac{X}{Y} - 1 \right| \leq 0.5 \right)$

(b) $P\left(Y \geq \frac{1}{2} \;\middle|\; Y \geq 1 - 2X \right)$

20% 4. Show that for a continuous random variable X with density function f and distribution function F, then

$$\mu = EX = \int_0^\infty [1 - F(x)]\, dx - \int_{-\infty}^0 F(x)\, dx \quad \text{if it exists.}$$

20% 5. Let X and Y be independent and exponentially distributed random variables with parameters λ and μ respectively. Calculate $P(X < Y)$.

20% 6. Let X_1 and X_2 be the numbers on two independent fair-die rolls. Let X be the minimum and Y be the maximum of X_1 and X_2. Calculate

(a) $E(Y \mid X = x)$

(b) $E(X \mid Y = y)$