(15%) 1. A sample size 2 is taken from the density function $f(x) = 1$, $0 < x < 1$, and zero elsewhere. What is the probability that $\overline{X}$ is greater than 0.9?

(15%) 2. Show that the random variables X_1 and X_2 with joint p.d.f. $f(x_1, x_2) = 12\, x_1 x_2 (1-x_2)$, $0 < x_1 < 1$, $0 < x_2 < 1$ and zero elsewhere. are stochastically independent.

(20%) 3. (a) If X is a nonnegative integer-valued random variable with $EX < \infty$. Show that $EX = \sum_{k=1}^{\infty} P\{X \geq k\}$.

(b) Let $A_1, A_2, \cdots, A_n$ be arbitrary events. Define
$$C_k = \{\text{at least } k \text{ of the } A_i \text{ occur}\}$$
Show that $\sum_{k=1}^{n} P(C_k) = \sum_{k=1}^{n} P(A_k)$

(20%) 4. (a) Define the moment generating function of a random variable X, when it exists.

(b) If $EX^k = 0.8$, $k = 1, 2, \cdots$.
Find the moment generating function of X and the probability mass function of X.

(15%) 5. Suppose that the joint density function of X and Y is given by
$$f(x, y) = \frac{e^{-x/y}\, e^{-y}}{y}, \quad 0 < x < \infty, \ 0 < y < \infty$$
Compute $E[X \mid y]$

(15%) 6. Let X be a continuous random variable with p.d.f. $f(x)$. Show that $f(\theta + x) = f(\theta - x)$ for all x if and only if $F(\theta + x) = 1 - F(\theta - x)$ for all x.