國立中央大學八十四學年度碩士班研究生入學試題卷

所別：數學研究所　　組　　科目：複變函數論　　共 1 頁 第 1 頁

1. Suppose that $z_1 + z_2 + z_3 = 0$, and $|z_1| = |z_2| = |z_3| = 1$. Show that z_1, z_2, z_3 are the vertices of an equilateral triangle. (15 %)

2. Find an analytic function $f(z) = u + iv$ on $z \neq 0$, such that $u = \frac{x^2 - y^2}{(x^2+y^2)^2}$. (15 %)

3. Find the linear fractional transformation $w = \frac{az+b}{cz+d}$ which carries the points $1, i, -i$ into $0, 1, -1$ respectively. (10 %)

4. Find the radius of convergence of the series $\sum_{n=0}^{\infty} [3 + (-1)^n]^n z^n$. (15 %)

5. Compute $\int_C \frac{3z-1}{z(z-1)}\,dz$, where C is the circle $|z| = 2$. (15 %)

6. Evaluate $\int_0^{\infty} \frac{\sin^2 x}{x^2}\,dx$. (15 %)

7. Let f be a function which is continuous in a neighborhood of the point a. Show that $\lim_{r \to 0} \int_{|z-a|=r} \frac{f(z)}{z-a}\,dz = 2\pi i f(a)$. (15 %)