1. (20%) Show that every real sequence with $lm+1$ terms contains either an increasing subsequence with $l+1$ terms or a decreasing subsequence with $m+1$ terms.

2. (20%) Show that there are at most $n-1$ orthogonal Latin squares of order $n \geq 2$.

3. (15%) Solve $h_n = \sum_{k=1}^{n-1} h_k h_{n-k}$ $n = 2, 3, \cdots$ and $h_1 = 1$

4. (10%) Show that every graph is an induced subgraph of a regular graph.

5. (10%) ① G is a planar graph with $n \geq 3$ vertices and e edges. Show that $e \leq 3n-6$.

(5%) ② G is a planar graph. Show that G contains a vertex with degree ≤ 5.

6. (5%) ① Express X^4 in the form $A_4\binom{x}{4} + A_3\binom{x}{3} + A_2\binom{x}{2} + A_1\binom{x}{1} + A_0\binom{x}{0}$

(5%) ② Express $\sum_{k=1}^{n} k^4$ as $B_5\binom{n+1}{5} + B_4\binom{n+1}{4} + B_3\binom{n+1}{3} + B_2\binom{n+1}{2}$

7. (5%) ① State the principle of inclusion and exclusion.

(5%) ② State Hall's Theorem (i.e. matching Theorem, marriage Theorem).