國立中央大學 111 學年度碩士班考試入學試題

所別： 數學系 碩士班 應用數學組(一般生)
數學系 碩士班 應用數學組(在職生)

科目： 微積分

Instructions: Do all problems. Show your work.

1. (10%) Find the limit $\lim_{n\to\infty}\sum_{k=1}^{n}\ln\sqrt[n]{1+\frac{k}{n}}$.

2. (10%) Find $f'(0)$ for $f(x)=\begin{cases} e^{-1/x^2}, & x\neq 0 \\ 0, & x=0. \end{cases}$

3. (10%) Find the values of p for which the integral $\int_1^2 \frac{dx}{x(\ln x)^p}$ converges.

4. (10%) Find $\frac{d^2y}{dx^2}$ if $x=\int_0^y \frac{1}{\sqrt{1+4t^2}}\,dt$.

5. (10%) Determine if the series $\sum_{n=1}^{\infty}\frac{1}{n(1+\ln^2 n)}$ converges or diverges.

6. (10%) Use the method of Lagrange multiplier to find the extreme values of $f(x,y,z)=xyz$ on the ellipse $x^2+2y^2+3z^2=1$.

7. (10%) Evaluate the integral $\int_0^1\int_{\sin^{-1}y}^{\frac{\pi}{2}}\cos x\sqrt{1+\cos^2 x}\,dxdy$.

8. (10%) Evaluate the integral $\int_0^{\infty} e^{-x^2}\,dx$.

9. (10%) Use the second derivative test to determine the relative extrema and saddle points of the function $f(x,y)=(x^2+y^2)e^{-y}$.

10. (10%) The surfaces $f(x,y,z)=x^2+y^2-2=0$ and $g(x,y,z)=x+z-4=0$ meet in an ellipse E. Find parametric equations for the line tangent to E at the point $P_0(1,1,3)$.