

國立中央大學 110 學年度碩士班考試入學試題

所別： 數學系 碩士班 應用數學組(一般生)
數學系 碩士班 應用數學組(在職生)

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科目： 微積分

本科考試禁用計算器

※計算題需計算過程，無計算過程者不予計分

*請在答案卷(卡)內作答

1.(10%) Find the derivative of the function $h(x, y) = \tan^{-1}(y/x) + \sqrt{3} \sin^{-1}(xy/2)$ at $(1, 1)$ in the direction $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j}$.

2.(10%) Find the value of the integral $\int_{5\pi/6}^{\pi} \frac{\cos^4 x}{\sqrt{1 - \sin x}} dx$.

3.(10%) Let $\Gamma(x)$ be the Gamma function defined by

$$\Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} dt \text{ for } x > 0.$$

Find the value of $\Gamma(5/2)$.

4.(10%) Let $f(x) : \mathbb{R} \rightarrow \mathbb{R}$ be an infinite differentiable function, $a, b \in \mathbb{R}$ are different constants. Show that there exists a number $\xi \in [a, b]$ such that

$$\int_a^b f''(x)(x - b) dx = -\frac{f''(\xi)}{2}(b - a)^2.$$

5.(10%) Find the interval of convergence of the series $\sum_{n=2}^{\infty} \frac{x^n}{n(\ln n)^2}$.

6.(10%) Let us define $f(x, y) = \begin{cases} xy \frac{x^2 - y^2}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0), \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$

Determine the partial derivatives $f_{xy}(0, 0) = f_{yx}(0, 0)$ or $f_{xy}(0, 0) \neq f_{yx}(0, 0)$.

7.(10%) Find the value of the double integral $\int_0^1 \int_0^1 e^{\max\{x^2, y^2\}} dy dx$.

8.(10%) Find the maximum and minimum values of $f(x, y, z) = 5 - z$ on the ellipse formed by the intersection of the cylinder $x^2 + y^2 = 8$ and the plane $x + y + z = 1$.

9.(10%) Find the value of the double integral

$$\int_0^2 \int_{y/2}^{(y+4)/2} y^3(2x - y)e^{(2x-y)^2} dx dy.$$

10.(10%) Find the area of the region bounded by the closed curve

$$C : \begin{cases} x(t) = 2 \cos t - \sin(2t), \\ y(t) = \sin t, \end{cases} \text{ for } t \in [0, 2\pi].$$