國立中央大學 107 學年度碩士班考試入學試題

所別: 數學系碩士班 數學組(一般生)

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科目: 高等微積分

本科考試禁用計算器

*請在答案卷(卡)內作答

Write legibly and logically. Decide how much details to include.

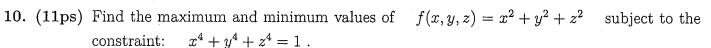
Part I: Definitions and Theorems. Complete the sentence in the case of a definition

- 1. (7ps) Let $n \geq 2$, $m \geq 1$ and $f: \mathbb{R}^n \to \mathbb{R}^m$. We say that f is differentiable at x_o if ...
- 2. (7ps) A set $X \subseteq \mathbb{R}^n$ is open if ...
- 3. (7ps) A set $X \subseteq \mathbb{R}^n$ is sequentially compact if...
- **4.** (7ps) Let $f, f_k : X \subseteq \mathbb{R}^n \to \mathbb{R}$ where k = 1, 2, ... We say that f_k converge **pointwise** to f on X if
- 5. (7ps) Let $f, f_k : X \subseteq \mathbb{R}^n \to \mathbb{R}$ where k = 1, 2, ... We say that f_k converge uniformly to f on X if
- 6. (7ps) State the Heine-Borel Theorem.
- 7. (7ps) State the Minimum-Maximum (also known as the Extreme Value) Theorem.

Part II: Computations

8. (10ps) Find
$$\lim_{x\to 0^+} \frac{\sqrt{\ln(\sec x)}}{x}.$$

9. (10ps) Find
$$\int_0^1 \int_{\sqrt{x}}^1 \sin\left(\frac{y^3+1}{2}\right) dy dx .$$



Part III: Proofs

- 11. (7ps) Let $f_k: X \subseteq \mathbb{R}^n \to \mathbb{R}$ be a sequence of continuous function. Suppose f_k converge uniformly to f on X. Prove that f is also continuous on X.
- 12. (3ps) Give an example showing (and write the proof for it) that this is not true if the f_k are just converging pointwise to f.
- 13. (10ps) True or false? If you think the following statement is false, give a counter-example (and prove that your example works) and if you think that it is true, prove it. Let $X \subset \mathbb{R}^n$ be an open set and $f: X \to \mathbb{R}$, $x_o \in X$, $\vec{n} \in \mathbb{R}^n$ is a unit vector. Suppose f is differentiable in every direction \vec{n} at x_o , then f is differentiable at x_o .

