國立中央大學103學年度碩士班考試入學試題卷

所別：數學系碩士班 計算數學組(一般生)　科目：微積分　共 / 頁 第 / 頁

本科考試禁用計算器　　＊請在試卷答案卷（卡）內作答

甲、計算、證明題：共 2 大題，每大題 10 分，共 20 分。須詳細寫出計算及證明過程，否則不予計分。

1. Evaluate (a) $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3+1}\,dxdy$ （5 分） (b) $\int \sin(\ln x)\,dx$ （5 分）

2. Does the function $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$ have a derivative at $x = 0$? Explain.

乙、填充題：共 10 題，每題 8 分，共 80 分。請將答案依題號順序寫在答案卷上，不必寫演算過程。

1. If $f(x) = x + e^x$, find $(f^{-1})'(1)$

Answer : _____

2. A 5-m long ladder learns against a wall. If the bottom of the ladder slides away from the wall at a rate of 1 m/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 3 m from the wall?

Answer : _____

3. Find the limit: $\lim_{h\to 0} \frac{1}{h} \int_x^{x+h} \sqrt{1+t^2}\,dt$

Answer : _____

4. Find the smallest value of $f(x,y) = x^2+2y^2-2x+3$ subject to the constraint $x^2+y^2 \leq 10$.

Answer : _____

5. Find the directional derivative of the function $g(x,y,z) = x + x\cos z - y\sin z + y$ at $(2,-1,0)$ if the point $P(x,y,z)$ move from $P_0(2,-1,0)$ toward the point $P_1(0,1,2)$?

Answer : _____

6. Find the length of the arc from $x = 0$ to $x = \pi/4$ for the curve $y = \int_0^x \sqrt{\cos 2t}\,dt$.

Answer : _____

7. Find the surface area of the cone $z = \sqrt{x^2+y^2}$, $0 \leq z \leq 2$.

Answer : _____

8. Evaluate $\int_{-1}^2 \frac{1}{x^3}\,dx$

Answer : _____

9. Evaluate $\int_C (2+x^2y)\,dx$, where C is the upper half of the unit circle $x^2+y^2 = 1$

Answer : _____

10. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the $x-$axis and the line $y = x - 2$.

Answer : _____