國立中央大學103學年度碩士班考試入學試題卷

所別：數學系碩士班 一般組(一般生)　科目：線性代數　共 2 頁 第 1 頁
數學系碩士班 一般組(在職生)
數學系碩士班 計算數學組(一般生)

本科考試禁用計算器　　*請在試卷答案卷（卡）內作答

Instructions: Do all 5 problems. Show your work. The notation, R, is the set of all real numbers

1. For each of the following subset of R^3, (i) determine whether it is a subspace of R^3. (10 pts)

   (a) $W_1 = \{(x_1, x_2, x_3) \in R^3 : x_1 + 3x_2 + 2x_3 = 0\}$.

   (b) $W_2 = \{(x_1, x_2, x_3) \in R^3 : x_1 + 3x_2 + 2x_3 = 4\}$.

   (c) $W_3 = \{(x_1, x_2, x_3) \in R^3 : x_1 x_2 x_3 = 0\}$.

   (d) $W_4 = \{(x_1, x_2, x_3) \in R^3 : x_1 + 2x_3 = 0, x_2 = 0\}$.

   (e) $W_5 = \{(x_1, x_2, x_3) \in R^3 : x_1 + 2x_3 = 0\}$.

   (ii) How about $W_1 \cup W_5$ and $W_1 \cap W_4$, are they subspaces of R^3? (5 pts). Justify your answer.

2. Let $T : R^3 \to R^3$ be given by

$$T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, -4x_1 + x_2 + 7x_3, -2x_1 - x_2 + 11x_3)$$

   (a) Show that T is a linear transformation. (10 pts)

   (b) Find a basis for the null space of T, the dimension of the null space, $dim(N(T))$ and the dimension of the range space, $dim(R(T))$. (10 pts)

3. Consider the following linear system,

$$Ax = b,$$

   where

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

   and

$$b = (4, 0, 0)^T$$

   (a) Use Gaussian elimination to solve the linear system for $x = (x_1, x_2, x_3)^T$ (10 pts)

   (b) Show that the matrix A is symmetric positive definite (10 pts)

參考用

注意：背面有試題

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4. Consider the matrix A given by

$$\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

(a) Find all eigenvalues of A and corresponding vectors (10 pts)

(b) Find an invertible matrix Q and a diagonal matrix D such that $D = Q^{-1}AQ$. (10 pts)

(c) Compute the determinant of A^{10}. Simplify your answer as much as possible.(5 pts)

5. On $P_2(R)$, consider the inner product given by

$$< p, q >= \int_0^1 p(x)q(x)\,dx$$

(a) Show that the basis $(1, x, x^2)$ is NOT orthonormal (10 pts)

(b) Apply the Gram-Schmidt procedure to $(1, x, x^2)$ to produce an orthonormal basis of $P_2(R)$. (10 pts)

Note that $P_2(R)$ is the set of all polynomials of degree 2 with real valued coefficients.

注意：背面有試題