

國立中央大學99學年度碩士班考試入學試題卷

所別：工業管理研究所碩士班 乙組(一般生) 科目：作業研究 共 2 頁 第 1 頁

本科考試禁用計算器

*請在試卷答案卷(卡)內作答

1. (15分) Let $n (n \geq 2)$ be an integer, A a square $n \times n$ matrix, and I an $n \times n$ identity matrix.

For example, if n is 3, then A could be $\begin{bmatrix} 10 & 0 & 43 \\ -26 & 3 & 12 \\ 0 & -15 & 4 \end{bmatrix}$ and I would be $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Let A^{-1} be an $n \times n$ matrix such that $AA^{-1} = A^{-1}A = I$, that is, A^{-1} is the *inverse* of A . Now, for any $n \times n$ matrix A , we can use the following method to find A^{-1} (if A^{-1} exists):

Form an $n \times 2n$ matrix $[A, I]$. Use the elementary row operations (矩陣基本列運算) to transform the left-half part of $[A, I]$ (that is, A in $[A, I]$) to become I . If we can make the transformation successfully, then the right-half part of the new $n \times 2n$ matrix would be A^{-1} .

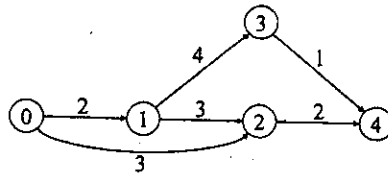
Please explain why we can use the above method to find A^{-1} .

2. (20分) Consider the following linear programming problem (P) where A is the coefficient matrix, x is the variable (column) vector and c is the cost (row) vector. The dual problem of (P), denoted as (D), can also be seen below.

$$\begin{array}{ll} (P): \text{Minimize } cx & (D): \text{Maximize } wb \\ \text{subject to } Ax \geq b & \text{subject to } wA \leq c \\ x \geq 0 & w \geq 0 \end{array}$$

Let x_0 be any feasible solution to the problem (P), that is, x_0 satisfies $Ax_0 \geq b$. And let w_0 be any feasible solution to the problem (D), that is, $w_0A \leq c$. Please use the information given so far to prove " $w_0b \leq cx_0$ " (you may already notice that this is the famous *weak duality property*).

3. (15分) An oil company wants to ship the maximum possible amount of oil (per hour) via pipeline from node 0 to node 4 in the following figure.



On its way from node 0 to node 4, oil must pass through some or all of stations 1, 2 and 3. The various arcs in the figure represent pipelines of different diameters. The number seen next to each arc is the arc capacity, that is, the maximum number of barrels of oil (millions of barrels per hour) that can be pumped through the arc.

The problem of determining the maximum number of barrels of oil per hour that can be sent from node 0 to node 4 can be formulated as a linear program. Please write down a complete formulation of such a linear program.

參考用

注意：背面有試題

國立中央大學99學年度碩士班考試入學試題卷

所別：工業管理研究所碩士班 乙組(一般生) 科目：作業研究 共 2 頁 第 2 頁

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*請在試卷答案卷(卡)內作答

4. (20分) At a tool service centre, the arrival rate is two per hour and the service potential is three per hour. The hourly wage paid to the attendant at the service centre is \$15 per hour and the hourly cost of a machinist away from his work is \$40.
- (5分) Calculate the average number of machinists being served or waiting to be served at any given time.
 - (5分) Calculate the average time a machinist spends waiting for service.
 - (5分) Calculate the total cost of operating the system for an 8-hour day.
 - (5分) Calculate the cost of the system if there were two attendants working together as a team (in effect acting together as a single server), each paid \$15 per hour and each able to service on average two customers per hour.
5. (20分) An outdoor café estimates that its profit depends on the weather conditions, which are classified into three states: sunny, cloudy, and rainy. Based on record, a transition matrix is shown:

		Tomorrow's Weather		
		Sunny	Cloudy	Rainy
Today's Weather	Sunny	.75	.20	.05
	Cloudy	.45	.40	.15
	Rainy	.35	.45	.20

- (10分) The café earns an average profit of \$120 on sunny days and \$40 on cloudy days, but it suffers an average loss of \$200 on rainy days. What is the expected daily profit over the upcoming week if the weather today is sunny?
 - (10分) The owner is considering selling his business, while businesses such as his are usually valued at six times their expected yearly profit (assume 365 days). What is a fair asking price for the business should the owner decide to sell it?
6. (10分) Suppose a test will correctly identify an illegal drug (禁藥) user as testing positive 99% of the time, and will correctly identify a non-user as testing negative 99% of the time. Assume SBL (超級籃球聯賽) decides to test all players for drug use, and that only 0.5% of the players actually use the drug. What is the probability that, given a player is tested positive, he is actually a drug user?

參考用

注意：背面有試題