第一題 (24 分, 每小題各 4 分)

請依據下列七項描述，判斷各為何種網路問題(Network Problem)，並且選擇適合該網路問題的數學模型(Mathematical Model)。例如，題 1 屬於網路問題 D，數學模型為丁，則題 1 的解答應該表示為(D, 丁)。

1. A retailer has six stores located in Chung-li area. Two stores (A, B) are running low on a product, while two stores (C, D) has access units and other two stores (E, F) are intermediate nodes with no access supply or demand of their own. This retailer wishes to transport the available products from (C, D) to (A, B), possibly through (E, F), at minimum total cost.

2. A trucking company must determine the least shipping cost to provide logistics service in a highway network.

3. Five different devices produced on five production lines, are needed to be inspected. The travel time of finished goods to inspection areas depends on both the production line and the inspection area. Management wishes to designate a separate inspection area to inspect the products such that the total travel time is minimized.

4. A chemical company produces chemicals which are held in a huge drum. A network of pipes and valves regulates the chemical flow from the drum to different production areas. The safety division must plan a procedure to empty the drum as fast as possible into a safety tub in the disposal area, using a network of pipes and valves.

5. Tao-Yuan County is planning the development of a new light rail transportation system. The system should link 10 residential and commercial centers. The county government needs to select the set of lines that will connect all the centers at a minimum total cost.

6. A manufacturer is planning its production for the next three months. Production capacity and unit production cost will change from month to month. The company can use both regular time and overtime to produce skis. Production levels should meet both demand forecasts and end-of-quarter inventory requirement. Management would like to schedule production to minimize its costs for the quarter.

7. Everyday every 7-11 truck must visit 15 local stores, going out from and returning to the same main office in Chung-li. The manager needs to find the cycle that minimizes the total cost required to visit all the stores exactly once.

Network Models

A. Shortest path problem
B. Maximum flow problem
C. Minimum spanning tree problem
D. Transshipment problem
E. Traveling Salesman Problem
F. Assignment problem
G. Transportation problem

Mathematical Models

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$$\begin{aligned}
\underset{x}{\text{minimize}} \quad & \sum_{(i,j)\in E} c_{i,j}x_{i,j} \\
\text{subject to} \quad & \sum_{(i,j)\in E} x_{i,j} = n-1 \\
& \sum_{(i,j)\in E\, i,j\in S} x_{i,j} \le |S|-1 \text{ for any node set } S \subseteq V \\
& x_{i,j} \in \{0,1\} \ (i,j) \in E
\end{aligned}$$

乙.

$$\begin{aligned}
\underset{x}{\text{minimize}} \quad & \sum_{i,j} c_{i,j}x_{i,j} \\
\text{subject to} \quad & \sum_{i} x_{i,j} = 1 \,\forall j \\
& \sum_{j} x_{i,j} = 1 \,\forall i \\
& x_{i,j} \in \{0,1\} \,\forall i,j
\end{aligned}$$

丙.

$$\begin{aligned}
\underset{x}{\text{minimize}} \quad & \sum_{i,j} c_{i,j}x_{i,j} \\
\text{subject to} \quad & \sum_{i} x_{i,j} \ge d_j \,\forall j \\
& \sum_{j} x_{i,j} \le s_i \,\forall i \\
& x_{i,j} \ge 0 \,\forall i,j
\end{aligned}$$

丁.

$$\begin{aligned}
\underset{x}{\text{minimize}} \quad & \sum_{i,j} c_{i,j}x_{i,j} \\
\text{subject to} \quad & \sum_{i} x_{i,j} \ge d_j \,\forall j \\
& \sum_{j} x_{i,j} \le s_i \,\forall i \\
& \sum_{i} x_{i,k} = \sum_{j} x_{k,j} \,\forall k \\
& x_{i,j} \ge 0 \,\forall i,j
\end{aligned}$$

注意：背面有試題

戊.

$$\underset{x}{\text{minimize}} \quad \sum_{(i,j)\in E} c_{i,j}x_{i,j}$$

$$\text{subject to} \quad \sum_{i|(i,k)\in E} x_{i,k} - \sum_{j|(k,j)\in E} x_{k,j} = \begin{cases} -1 & k = s \\ 1 & k = t \\ 0 & \text{otherwise} \end{cases}$$

$$x_{i,j} \in \{0,1\} \ (i,j) \in E$$

E is the set of edges in the underlying graph G = (V, E).

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$$\underset{x}{\text{maximize}} \quad f$$

$$\text{subject to} \quad \sum_{i|(i,k)\in E} x_{i,k} - \sum_{j|(k,j)\in E} x_{k,j} = b_k \ \forall k$$

$$b_k = \begin{cases} -f & k = s \\ f & k = t \\ 0 & \text{otherwise} \end{cases}$$

$$0 \le x_{i,j} \le u_{i,j}$$

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$$\underset{x}{\text{minimize}} \quad \sum_{i,j|i\neq j} c_{i,j}x_{i,j}$$

$$\text{subject to} \quad \sum_{j|j\neq i} x_{i,j} = 1 \ \forall i$$

$$\sum_{i|i\neq j} x_{i,j} = 1 \ \forall j$$

$$x_{i,j} \in \{0,1\}$$

第二題 (Total 10 分)

Given

D = annual demand for product X　C = purchase cost/per unit　Q = economic order quantity

C_o= cost of placing an order　C_h = annual holding cost/per unit of average inventory

d = average daily demand　L = lead time (days)

(1) Draw a graph showing the trade-off decision of this EOQ problem (3 分).

(2) Formulate the "Total Annual Inventory Cost" (including annual holding cost, annual ordering cost, and annual purchase cost) with defined notations (4 分).

(3) Formulate the Reorder Point (3 分).

參考用

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第三題 (Total 16 分)

Two facilities (F3 and F4) of NCU Co. supply three client companies (C5, C6, and C7) with customized furniture for their offices. They both order furniture from the same two manufacturers (M1 and M2). Currently weekly demands by the client companies are 50 for C5, 60 for C6, and 40 for C7. Both manufacturers (M1 and M2) can supply at most 75 units, whereas unit costs from the manufacturers to the NCU facilities (F3 and F4) are:

	F3	F4
M1	5	8
M2	7	4

The costs to install the furniture at the clients' locations are:

	C5	C6	C7
F3	1	5	8
F4	3	4	4

(1) Draw a network of the problem (8 分).

(2) Formulate a Linear Programming model to minimize the overall cost (8 分).

Decision variables are defined:

X_{ij} = amount shipped from manufacturer i to facility j

X_{jk} = amount shipped from facility j to client company k; where $i = 1, 2; j = 3, 4; k = 5, 6, 7$

第四題 (15 分)

Let X_n be the sum of the first n outcomes of tossing a six-side die repeated in an independent fashion. Compute

$$\lim_{n\to\infty} P(X_n \text{ is divisble by } 7).$$

第五題 (20 分)

Consider a queuing network which consists of two stations in series. Customers come to the first station from outside with respect to a Poisson process with rate λ. Service times at station 1 are independent identically distributed with rate μ_1 and the service times at station 2 are independent identically distributed with rate μ_2. Let $\pi_{n,m}$ be the long probability of having n customers at station 1 and m customers at station 2. If $\lambda < \min\{\mu_1, \mu_2\}$, verify that

$$\pi_{n,m} = \left(\frac{\lambda}{\mu_1}\right)^n \left(1-\frac{\lambda}{\mu_1}\right)\left(\frac{\lambda}{\mu_2}\right)^m \left(1-\frac{\lambda}{\mu_2}\right).$$

第六題 (15 分)

You start with one dollar. A fair coin is tossed 25 times. Each time the coin comes up head, your fortune doubles. Each time the coin comes up tails. Your fortune is reduced by one half. Let X be your fortune after all 25 tosses.

Find $P(X=2)$

注意：背面有試題