所別：工業管理研究所碩士班 乙組 科目：作業研究

1. (35 points) At each play of the game, a gambler can bet any nonnegative amount up to his present fortune and will either win or lose that amount with probabilities p and $q=1-p$, respectively. The gambler is allowed to make n bets and he is allowed to bet a fraction, $0\le\alpha\le 1$, of his present fortune at each bet. His objective is to maximize the expectation of the logarithm of his final fortune. Let $V_n(x)$ denote the maximal expected return if the gambler has a present fortune of x and is allowed n more gambles and $V_0(x)=\ln x$.
   (a) (15 points) Suppose $p>1/2$. Using $V_0(x)=\ln x$, show that $V_1(x)=c+\ln x$, where $c=\ln 2+p\ln p+q\ln q$.
   (b) (10 points) Suppose $p>1/2$. Show that $V_n(x)=nc+\ln x$, for all n and the optimal strategy is always to bet the fraction $p-q$ of his present fortune.
   (c) (10 points) When $p\le 1/2$, show that $V_n(x)=\ln x$, for all n, and the optimal strategy is always to bet 0.

2. (15 points) Consider an urn model Markov chain described as follows. There are M total molecules distributed among two urns. At each time n, one of the molecules is chosen at random, removed from its urn, and place in the other urn. Let X_n denote the number of molecules in Urn 1 at time n. Show that

$$E[X_n]=\frac{M}{2}+\left(\frac{M-2}{M}\right)^n\left(E[X_0]-\frac{M}{2}\right).$$

3. (15 points) Consider the following LP:

Maximize $z=2x_1+4x_2+4x_3-3x_4$

Subject to

$$x_1+x_2+x_3=4$$
$$x_1+4x_2+x_4=8$$
$$x_1,x_2,x_3,x_4\ge 0$$

Use the dual problem to verify that the basic solution (x_1,x_2) is not optimal.

4. (15 points) Show that the dual of

$$\textbf{Maximize } z=\{\mathbf{CX}\mid \mathbf{AX}\le\mathbf{b},\ \mathbf{0}<\mathbf{L}\le\mathbf{X}\le\mathbf{U}\}$$

always possesses a feasible solution.

5. (20 points) ABC Company produces four parts that require the use of a lathe and a drill press. The two machines operate 10 hours a day. The following table provides the time in minutes required by each part:

	Production time in min	
Part	*Lathe*	*Drill press*
1	5	3
2	6	2
3	4	6
4	7	4

It is desired to balance the two machines by limiting the difference between their total operation times to at most 30 minutes. The market demand for each part is at least 10 units. Additionally, the number of units of part 1 may not exceed that of part 2. Formulate the problem as a goal programming model.