國立中央大學 110 學年度碩士班考試入學試題

所別： 工業管理研究所 碩士班 不分組(一般生) 共4頁 第1頁

科目： 作業研究

本科考試禁用計算器 ＊請在答案卷(卡)內作答

1. (35% in total)

Consider this situation which could become real in the near future: We have a number of predetermined (事先決定的) locations from which 5G signals will be emitted (發射) by using air-borne autonomous drones (空中無人機). Of course, we wish to cover the entire area with 5G. There are a number of companies manufacturing various drone models. These drone models have different weight-lifting (載重) capabilities, which means they can carry different 5G equipment into the air to cover a different area size. Naturally, if a drone model can carry more weight, it is also more expensive to purchase. Moreover, for those predetermined locations to emit 5G signals, each of them can only deploy some of the drone models due to various reasons/considerations (that is, a drone model that is OK for one location may not be OK for another location).

In the following, we will try to develop a mixed-integer programming (MIP) model for the above problem. Our objective is to determine a specific drone model for each location with the lowest total costs, while making sure 5G signals will cover the entire area. Let us first define subscripts (下標) for this MIP model.

i, j a particular drone model

m, n a particular location

Next, let us define parameters (參數，亦即「已知資訊」) for things described in the above.

locations the set of all locations to emit 5G signals

$\mathbf{drones}_m$ the set of all drone models which are qualified for use at location m (能夠在 location m 使用所有 drone models 的集合; each location in **locations** has such a drone model set so we know which drone models we can consider)

$\mathbf{radius}_i$ the maximum radius that drone model i can reach for effective coverage of 5G signals (drone model i 所能提供 5G 信號有效覆蓋的最大半徑)

$\mathbf{costs}_{im}$ the total costs of using drone model i to emit 5G signals at location m (note that for some drone models, their total costs may be different from one location to another)

$\mathbf{distance}_{mn}$ the distance between location m and location n

Finally, the following are decision variables for this MIP model.

X_{im} binary (0/1) decision variable which equals 1 if drone model i is used at location m to emit 5G signals, and 0 otherwise

D_m, D_n nonnegative decision variables which represent the radius of 5G coverage at location m or n

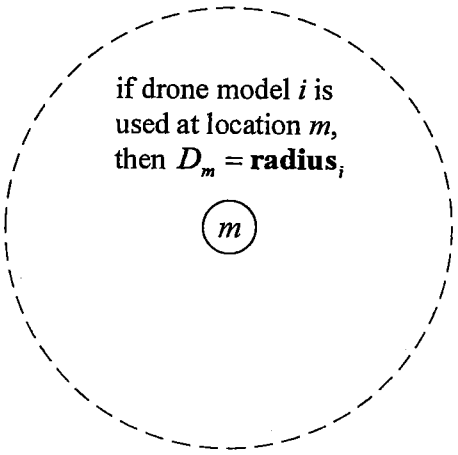
注意:背面有試題

In the following, we will develop this MIP model one equation at a time. **Please note**: We will only use the above already defined subscripts, sets, parameters, and decision variables. In other words, "在模型中禁止使用任何自行定義的符號，無論是下標、參數、決策變數等". **Please also note**: We only want to develop a linear MIP model (必需是線性 MIP 模型).

First, since each location must have a drone model (which is OK for that location) to send 5G signals, we obviously will need this constraint in the model: $\sum_{i \in \mathbf{drones}_m} X_{im} = 1, \ \forall m \in \mathbf{locations}$.

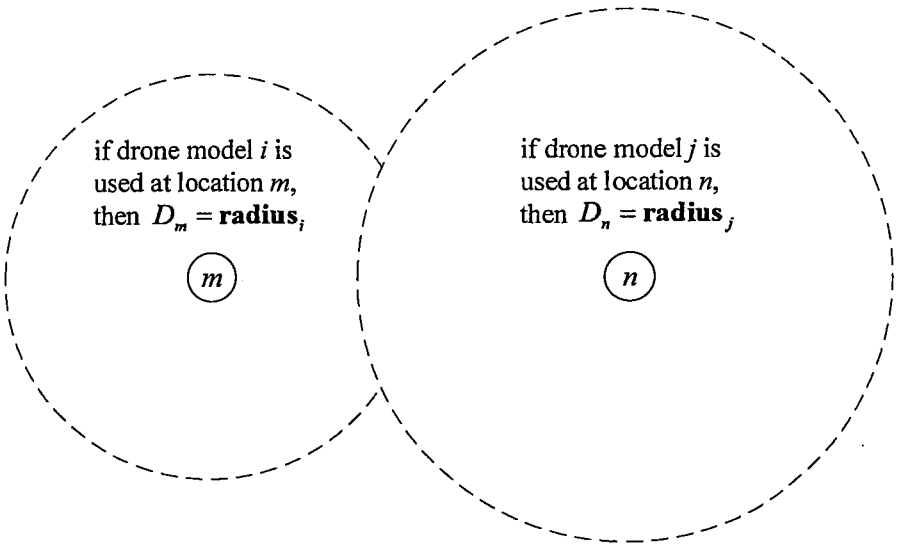
Question 1.1 (10%)

Next, the drone model chosen for each location will determine that location's effective 5G-covering area. Shown in the following figure, if drone model $i \in \mathbf{drones}_m$ is chosen for location m, then D_m will equal $\mathbf{radius}_i$; but if a different drone model in $\mathbf{drones}_m$ is chosen, D_m could have a different value. For each location m in the **locations** set, please develop an equation to determine the real value of D_m.



Question 1.2 (10%)

To make sure that 5G signals will cover the entire area, we must enforce a specific overlap (特定程度的重疊) on the two areas covered by any two nearby locations. Shown in the following figure, if m and n are two such locations, then the sum of D_m and D_n must be at least 1.5 times of the distance between the two locations (no matter the drone model used in each location to emit 5G signals). Please develop an equation to enforce that requirement in the model.



Question 1.3 (15%)

Finally, please develop an objective function for the MIP model that will minimize the total costs to choose an appropriate drone model for every location in the **locations** set.

2. (15% in total)

Consider the following linear system with its coefficient matrix **A** given right next to it.

$$\begin{aligned} &x_1 + x_4 = 4 \\ &x_2 + x_5 = 4 \\ &x_1 + x_2 + x_6 = 6 \\ &x_1 - 2x_3 - x_7 = -4 \\ &x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0 \end{aligned}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & -2 & 0 & 0 & 0 & -1 \end{bmatrix}$$

What is the basic feasible solution corresponding to the first four columns of **A**? That is, use the first four columns of **A** as the basis to find a basic feasible solution (求解 **A** 的前四行所對應到的基本可行解). **Note**: You must use linear algebra related mathematics to find your answer (求解必需使用與「線性代數相關」的數學計算，否則不予計分).

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3. (24%)

Two players are tossing (possibly biased) coins, on each toss, the probability player 1 wins one cent is p, and the probability player 1 loses one cent is $q = 1 - p$, where c is the total number of pennies of both players. Define a Markov chain $\{X_n\}$, where $X_n = j$ means that player 1 has j cents after the n-th toss. The game continues until one player goes broke (the other player wins).

(a) What are the absorbing states of this Markov chains? (4 points)

(b) What is the transition probability matrix of this chain? (10 points)

(c) Let $T = \{1, \ldots, c-1\}$ is a finite set of transient states and x_j is the probability that player 1 wins given $X_0 = j, j \in T$. Write down the systems of equations that x_j need to satisfy. (10 points)

4. (26%)

(a) (6 points) Consider two independent M/M/1 queues, the first one with arrival rate λ and service rate μ_1; and the second one with arrival rate λ and service rate μ_2. What is the probability that there are n customers at server 1? What is the probability that there are m customers at server 2 ?

(b) Consider a two-server system in which customer inter-arrival times are exponentially distributed with rate λ at server 1. After being served by server 1, customers then join the queue for server 2. We assume that there is infinite waiting space at both servers. Each server serves one customer at a time with server i taking an exponential time with rate μ_i for a service, $i = 1, 2$. To analyze this system, we need to keep track of the number of customers at server 1 and the number of customers at server 2. Let define the state by pair (n, m)—meaning that there are n customers at server 1 and m customers at server 2 and $P_{n,m}$ denote the probability of being in that state.

(b-1) (10 points) What are the balance equations for this system?

(b-2) (5 points) If the number of customers at server 1 and server 2 were independent random variables, what would be the expression for $P_{n,m}$?

(b-3) (5 points) Verify that your solution satisfies the balance equations from (b-1).

注意:背面有試題