國立中央大學 110 學年度碩士班考試入學試題

所別： 工業管理研究所 碩士班 不分組(一般生) 共 4 頁 第 1 頁

科目： 統計學 ※計算題需計算過程，無計算過程者不予計分

本科考試可使用計算器，廠牌、功能不拘 ＊請在答案卷(卡)內作答

1. If X is a random variable with Pareto distribution, then the probability that X is greater than some number x, i.e. the complementary CDF, is given by

$$F^c(x) = \begin{cases} \frac{x_m^\alpha}{x}, x \geq x_m \\ 1 \quad , x < x_m \end{cases}$$

, where x_m is the (necessarily positive) minimum possible value of X, and α is a positive parameter.

(a) What is the pdf of X? (5 points)

(b) Determine $E[X]$ for $\alpha > 1$. (10 points)

(c) What is the moment generating function of X, $E[e^{tX}]$, where $t \leq 0$? (10 points)

Hint: Use incomplete gamma function: $\Gamma(\alpha, u) = \int_u^\infty t^{\alpha-1}e^{-t}dt$

(d) Let X_1 and X_2 are independent identically distributed random variables with Pareto distribution with parameter x_m and α defined earlier. Show that the two random variables $\min\{X_1, X_2\}$ and $(X_1 + X_2)/\min\{X_1, X_2\}$ are independent. (10 points)

2. (15 points) Let X and Y be continuous random variables having a joint density. Suppose that Y and $\varphi(X)Y$ have finite expectation. Show that

$$E[\varphi(X)Y] = \int_{-\infty}^{\infty} \varphi(x)E[Y|X = x]f_X(x)dx.$$

注意:背面有試題

所別： 工業管理研究所 碩士班 不分組(一般生)　　共4頁 第2頁

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3. Two machines are used for filling plastic bottles with a net volume of 650 ml. The filling processes can be assumed to be normal, and the two standard deviations are unknown but equal. The quality engineering department suspects that whether or not the filling volume is 650 ml. An experiment is performed by taking a random sample of 10 bottles from each machine.

Machine 1		Machine 2	
653	651	652	653
654	646	647	654
655	648	646	652
655	652	651	651
652	649	649	650

(a) Assume the mean filling volumes of two machines are the same. Please state the hypotheses that should be tested in this experiment. (2 points)

(b) Test these hypotheses using $\alpha = 0.05$. What are your conclusions? (10 points)

(c) Find the p-value for this test. (3 points)

4. Follow problem 3, if the quality engineering department suspects that both machines fill to the same net volume, whether or not this volume is 650 ml. In other words, the mean filling volumes of two machines might be different.

(a) Please use t-test on the equality of mean filling volumes of two machines at $\alpha = 0.1$. What are your conclusions? (10 points)

(b) Find a 90 percent confidence interval on the difference in mean fill volume for the two machines. (5 points)

(c) Rework the problem/test via the analysis of variance (ANOVA) approach. Prepare an ANOVA table to summarize the results. Show why the conclusion is coincidence to (a). (Hint: the corresponding critical F-value can be calculated from a t-value.) (10 points)

5. Consider the test in Problem 4. If the mean fill volumes of the two machines differ by as much as 5 ml, what is the power of the test used in Problem 4. (give a range based on the provided t-table) (5 points) What sample size would result in a power of at least 0.95 if the actual difference in mean fill volume is 5 ml? (5 points)

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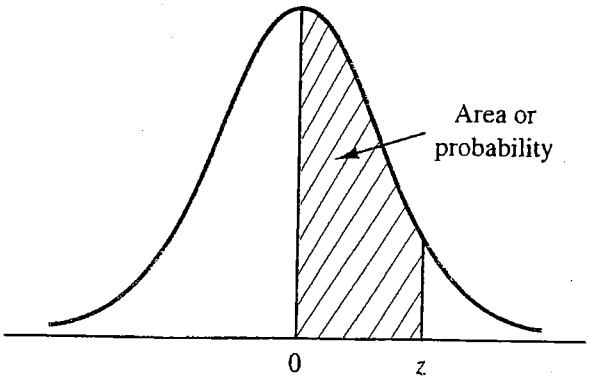

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean and z is .3944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4986	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

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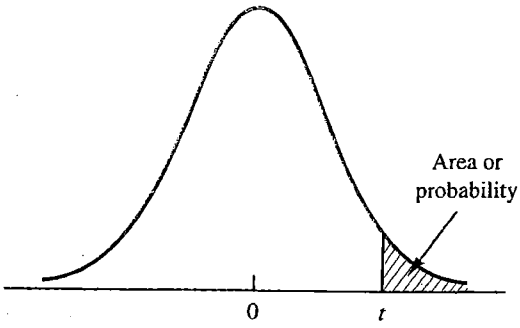

Entries in the table give t values for an area or probability in the upper tail of the t distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail, $t_{.05} = 1.812$.

Degrees of Freedom	Area in Upper Tail				
	.10	.05	.025	.01	.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

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