國立中央大學 109 學年度碩士班考試入學試題

所別： 工業管理研究所 碩士班 不分組(一般生)

科目： 統計學

本科考試可使用計算器，廠牌、功能不拘 ＊請在答案卷(卡)內作答

＊計算題需計算過程，無計算過程者不予計分

1. In probability, the characteristic function of random variable X is defined as follows,

$$\phi_X(t) = E\left[e^{itX}\right],$$

where $t \in R$ and $i = \sqrt{-1}$ is the imaginary unit.

(a) (10 points) Determine the characteristic function of the standard normal random variable.

(b) (5 points) Determine the characteristic function of the continuous uniform random variable whose value is in between -1 and 1.

(c) (5 points) Continue from (b). Determine the characteristic function of the sum of two independent continuous uniform random variable whose value is in between -1 and 1.

(d) (5 point) Determine the moment generating function $M(t)$ of the sum of two independent continuous uniform random variable whose value is in between -1 and 1 from the solution of (c).

2. Consider two independent $U(0,1)$ random variable X_1 and X_2.

(a) (5 points) Determine the CDF of X_1.

(b) (5 points) Let $Y = X_1 + X_2$. Determine the conditional CDF of Y given $X_2 = x$, where $0 \le x \le 1$.

(c) (5 points) Continue from (b). Determine the CDF of Y.

(d) (10 points) Let $Z = \min\{X_1, X_2\}$. Determine the CDF of Z.

3. (20 points) To estimate the population mean, $\boldsymbol{\mu}$, of a particular population, a ***random sample*** of size **5** is drawn. Let X_1, X_2, X_3, X_4, X_5, denote the observations. Andy, Betty, Cathy, David, and Eddy propose different estimators of $\boldsymbol{\mu}$ as following:

(A)ndy: Sample mean, $\frac{X_1+X_2+X_3+X_4+X_5}{5}$;

(B)etty: Weighted sample mean, $\frac{X_1+2X_2+3X_3+4X_4+5X_5}{15}$;

(C)athy: Median, $X_{(3)}$;

(D)avid: Weighted sample mean by order statistics, $\frac{X_{(1)}+2X_{(2)}+3X_{(3)}+2X_{(4)}+X_{(5)}}{9}$;

(E)ddy: Truncated sample mean, $\frac{X_{(2)}+X_{(3)}+X_{(4)}}{3}$.

參考用

Please compare these proposed estimators and **Rank** them based on unbiasedness, efficiency, consistency, and etc. Try to show your idea to support your comparison as much as possible, (for example, sufficient statistics,… etc.)

注意：背面有試題

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4. Following (3), assume the population follows a normal distribution. Frank has drawn a sample and provided a 95% confidence interval of (75, 95), Grace has drawn another sample and provided a 95% confidence interval of (72, 88), and Hector has drawn another sample and provided a 95% confidence interval of (73, 95).
   (a) (5 points) What is the probability that **all** these three intervals cover the parameter μ? Assume μ has a prior distribution of normal with mean 83 and variance 25.
   (b) (10 points) Let's pool all observations of these three samples together, and please provide a pooled 95% confidence interval of μ. What is the probability of this pooled confidence interval covers μ.

5. Assume that the birthweight in grams of a baby born in Taiwan is $N(3315,525^2)$, boys and girls combined. Let X be the weight of a baby girl who is born at home in ChungLi city and assume that the distribution of X is $N(\mu_X,\sigma_X^2)$.
   (a) (5 points) Using 11 observations of X, **define a test statistic** and **critical region**, $\alpha = 0.01$, for testing $H_0: \mu_X = 3315$ against the alternative hypothesis $H_a: \mu_X > 3315$ (home-born babies are heavier).
   (b) (5 points) Calculate the value of the test statistic and give your conclusion using the following weights:

   3120 2657 3459 3629 3345 3629 3515 3856 3629 3345 3062

   (c) (5 points) Let's assume the standard deviation of X is the same as the standard deviation of birthweight of a newborn baby in Taiwan. Please redo (a) and (b).

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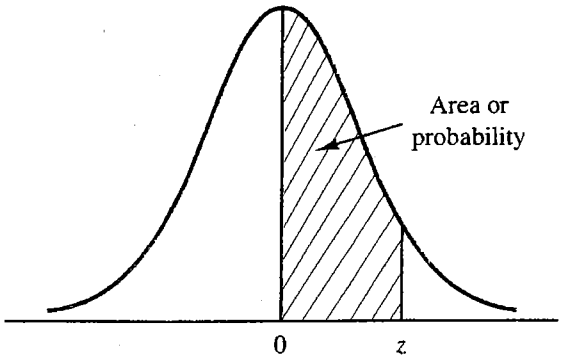

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean and z is .3944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4986	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

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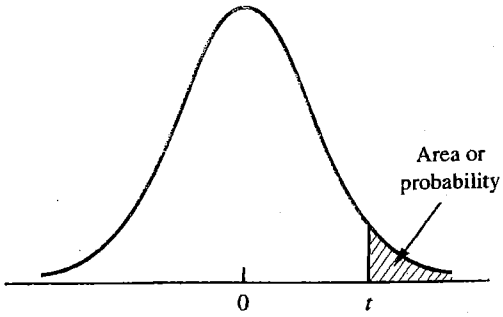

Entries in the table give t values for an area or probability in the upper tail of the t distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail, $t_{.05} = 1.812$.

Degrees of Freedom	Area in Upper Tail				
	.10	.05	.025	.01	.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

參考用

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