國立中央大學107學年度碩士班考試入學試題

所別： 工業管理研究所 碩士班 不分組(一般生)　　

科目： 統計學

本科考試可使用計算器，廠牌、功能不拘　　*請在答案卷(卡)內作答

1. Let X_1, X_2 be i.i.d. r.v.'s distributed as $U(\alpha, \beta)$. Set $Y_1 = X_1 + X_2$ and $Y_2 = X_2$
   (a) Find the joint pdf of X_1 and X_2 (5 points)
   (b) Find the joint pdf of Y_1 and Y_2 (5 points)
   (c) Find the pdf of Y_1 (10 points)

2. Let X be an r.v. distributed as $U(\alpha, \beta)$.
   (a) (10 points) Show that its m.g.f. is given by
   $$M(t) = \frac{e^{t\beta} - e^{t\alpha}}{t(\beta - \alpha)}$$
   (b) (10 points) By differentiation, show that $E[X] = (\alpha + \beta)/2$ and $Var(X) = (\beta - \alpha)^2/12$

3. Let Y be a nonnegative r.v. with CDF F_Y. It can be shown that
   $$E[Y] = \int_0^\infty (1 - F_Y(u))du$$
   Show that for a r.v. X with both positive and negative values,
   $$E[X] = E[X_+] - E[X_-] = \int_0^\infty (1 - F_X(x))dx + \int_{-\infty}^0 F_X(x)dx$$
   where $X_+ = Max\{X, 0\}$ and $X_- = Max\{-X, 0\}$. (10 points)

4. To test on the mean parameter of a Poisson distribution, λ, we have formed the null hypothesis, H_0: $\lambda = 1$, and the alternative hypothesis H_a: $\lambda = 2$. Let X_1, X_2, and X_3 be a random sample of size 3 from this particular Poisson distribution, and the rejection rule is set to be rejecting H_0 if the observed sum $\sum_{i=1}^{3} x_i \geq 6$.
   (a) (10 pts) Determine the significance level, α, of the test.
   (b) (10 pts) Find the power function, $\kappa(\lambda)$, of the test.
   (c) (5 pts) Determine the probability of type II error, β, of the test corresponding to the alternative hypothesis, H_a: $\lambda = 2$.

5. Drunk driving is a serious problem. It is believed that the percentage of drinking and driving in Taiwan is more than 20%. A random sample of 5 drivers were checked in a routine spot check.
   (a) (5 pts) Let $\bar{P}$ be the sample proportion of drunk driving. Please show the sampling distribution of $\bar{P}$. (Note that the normal approximation cannot be applied here.)
   (b) (10 pts) If 3 drivers are found drunk driving, (in other words, $\bar{p} = 0.6$), try to develop a 95% confidence interval of the population proportion of drunk driving. What is your conclusion of testing H_0: $p = 0.2$ at $\alpha = 0.05$.

注意:背面有試題

參考用

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(c) (10 pts) After collecting 10 samples, (in other words, 10×5 drivers in total), we found 3, 0, 1, 0, 1, 1, 1, 0, 4, and 2 drunk drivers respectively in each samples. Let's pool all the records together to develop a 95% confidence interval of the population proportion of drunk driving. What is your conclusion of testing H_0: $p = 0.2$ at $\alpha = 0.05$. (Note that the normal approximation can be applied here.)

The following table presents selected Poisson distributions with parameter $\lambda = 0.5, 1, 1.5, \ldots, 10$. The probabilities tabled are

$$P(X \leq x) = \sum_{k=0}^{x} e^{-\lambda} \frac{\lambda^k}{k!}$$

	$\lambda = E(X)$																			
x	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10
0	0.6065	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067	0.0041	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001	0
1	0.9098	0.7358	0.5578	0.4060	0.2873	0.1991	0.1359	0.0916	0.0611	0.0404	0.0266	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008	0.0005
2	0.9856	0.9197	0.8088	0.6767	0.5438	0.4232	0.3208	0.2381	0.1736	0.1247	0.0884	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042	0.0028
3	0.9982	0.9810	0.9344	0.8571	0.7576	0.6472	0.5366	0.4335	0.3423	0.2650	0.2017	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149	0.0103
4	0.9998	0.9963	0.9814	0.9473	0.8912	0.8153	0.7254	0.6288	0.5321	0.4405	0.3575	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403	0.0293
5	1	0.9994	0.9955	0.9834	0.9580	0.9161	0.8576	0.7851	0.7029	0.6160	0.5289	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885	0.0671
6	1	0.9999	0.9991	0.9955	0.9858	0.9665	0.9347	0.8893	0.8311	0.7622	0.6860	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649	0.1301
7	1	1	0.9998	0.9989	0.9958	0.9881	0.9733	0.9489	0.9134	0.8666	0.8095	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687	0.2202
8	1	1	1	0.9998	0.9989	0.9962	0.9901	0.9786	0.9507	0.9319	0.8944	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918	0.3328
9	1	1	1	1	0.9997	0.9989	0.9967	0.9919	0.9829	0.9682	0.9462	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218	0.4579
10	1	1	1	1	0.9999	0.9997	0.9990	0.9972	0.9933	0.9863	0.9747	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453	0.5830
11	1	1	1	1	1	0.9999	0.9997	0.9991	0.9976	0.9945	0.9890	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520	0.6968
12	1	1	1	1	1	1	0.9999	0.9997	0.9992	0.9980	0.9955	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364	0.7916
13	1	1	1	1	1	1	1	0.9999	0.9997	0.9993	0.9983	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981	0.8645
14	1	1	1	1	1	1	1	1	0.9999	0.9998	0.9994	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400	0.9165
15	1	1	1	1	1	1	1	1	1	0.9999	0.9998	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665	0.9513
16	1	1	1	1	1	1	1	1	1	1	0.9999	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823	0.9730
17	1	1	1	1	1	1	1	1	1	1	1	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911	0.9857
18	1	1	1	1	1	1	1	1	1	1	1	1	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957	0.9928
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.9999	0.9997	0.9995	0.9989	0.9980	0.9965
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.9999	0.9998	0.9996	0.9991	0.9984
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.9999	0.9998	0.9996	0.9993
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.9999	0.9999	0.9997
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.9999	0.9999
24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

參考用

注意:背面有試題