國立中央大學 106 學年度碩士班考試入學試題

所別： 工業管理研究所 碩士班 不分組(一般生)　　共 2 頁 第 1 頁

科目： 統計學

本科考試可使用計算器，廠牌、功能不拘　　＊請在答案卷　內作答

須有計算過程

1. Consider two nonnegative random variables W and X, where the pdf of X, f_X, is given and with known E[X] and Var(X). The pdf of W, however, is unknown to us but with following two properties. The first property is the pdf of W, $f_W(w)$, is proportional to the magnitude of itself, w. The second is that $f_W(w)$ is also proportional to $f_X(w)$
   (a) (10 pts) Derive $f_W(w)$ in terms of f_X
   (b) (5 pts) Determine the pdf of W if $X \sim \text{Exp}(\lambda)$
   (c) (10 pts) Determine the expected value of W

2. Consider a random variable X with CDF $F(x) = 1 - exp(-\frac{x^2}{2\theta})$, $x > 0$, i.e. Rayleigh distribution (X/θ follows chi-squared distribution with 2 degree of freedom).
   (a) (5 pts) Find the inverse CDF of X. (Set $u = F(x)$, solve for x)
   (b) (10 pts) Show that MLE of θ is $\hat{\theta} = \frac{1}{2n}\sum_{i=1}^{n} x_i^2$ (Hint: Find the pdf of X first and then determine the maximizer of the log likelihood function)
   (c) (10 pts) Determine the distribution of $\hat{\theta}$ as function of chi-squared distribution.

3. (20 pts) Fit $y = a + x + \varepsilon$ to the data

x	0	1	2
y	1	3	4

   where ε is assumed to follow normal distribution with mean 0 and variance σ^2. Find L.S.E. (least squares estimates) and M.L.E. (maximum likelihood estimates) of a and σ^2, respectively.

4. Suppose we have 5 observations as following:

   7, 6, 5, 5, 3.

   Assume they are taken from a Normal distribution with mean μ and variance σ^2.
   (a) (5 pts) If σ^2 is unknown. Provide **90%** confidence interval of μ.
   (b) (5 pts) Assume σ^2 is known as 2. Provide **99%** confidence interval of μ.
   (c) (20 pts) Assume σ^2 is known as 2. Suppose the unknown μ has a prior distribution as $N(5, 1)$. Based on the 5 observations above, what is the posterior distribution of μ?

參考用

注意：背面有試題

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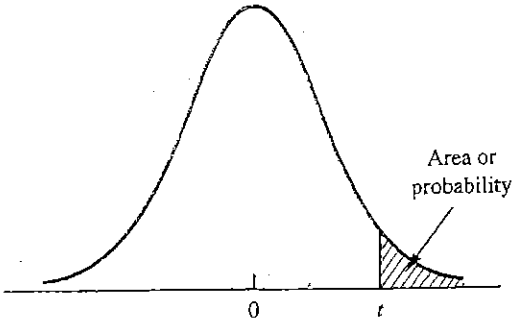

Entries in the table give *t* values for an area or probability in the upper tail of the *t* distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail, $t_{.05} = 1.812$.

Degrees of Freedom	Area in Upper Tail				
	.10	.05	.025	.01	.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576