1. (20%) Please find the following limit

(a) (10%) \[ \lim_{x \to 0} \frac{x^2}{e} \int_0^1 e^{x-y} \, dy \]

(b) (10%) \[ \lim_{(x,y) \to (0,0)} \frac{1 - \cos(x^2 + y^2)}{x^2 + y^2} = ? \]

2. (10%) Let \( f \) be a twice differentiable function with \( f(1) = 2, \ f'(1) = 0 \) and \( f''(x) > 0 \) for \( 0 < x < 1 \). Prove that \( f(0) > 1 \).

3. (15%) \[ \lim_{n \to \infty} \sum_{n=1}^{\infty} \frac{3}{2n + 3t - 2} = ? \]

4. (20%) Please evaluate the following integrals.

(a) (10%) Given \( |x| < 1 \), find \( \int_1^{1-x} \frac{1}{x^2} \, dx = ? \)

(b) (10%) Find the following integral by reversing the order: \( \int_0^1 \int_0^x e^{x+y} \, dy \, dx = ? \)

5. (15%) Let \( \Gamma(x) = \int_0^\infty t^{x-1} e^{-t} \, dt \), \( x > 0 \).

(1) (10%) Prove that \( \Gamma(x+1) = x\Gamma(x) \)

(2) (5%) Show that \( \Gamma\left(\frac{1}{2}\right) = 2\int_0^\infty e^{-\frac{1}{2}x} \, dx \)

6. (10%) Solving the following ODE: \( y'' = \frac{e^x - \cos x}{2y} \), \( y(0) = 2 \).

7. (10%) The call option price at current time \( t=0 \) is \( C_0 = S_0 N(d_1) - Ke^{-rT} N(d_2) \),

where \( d_1 = \frac{\ln \left( \frac{S_0}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}} \), and \( d_2 = d_1 - \sigma \sqrt{T} = \frac{\ln \left( \frac{S_0}{K} \right) + \left( r - \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}} \); \( r, \sigma, T, \) and \( K \) are constant. The function \( N(x) \) is the probability that a standard normal random variable is less than \( x \). That is, \( N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} y^2} \, dy \). Please prove that \( \frac{\partial C}{\partial S_0} = N(d_1) \).