1. (15%) The driver-training school gives a pretest to graduates before they take the official driving test. Sixty percent of the graduates pass a pretest, while the other 40 percent fail. Of the graduates who pass the pretest on the first attempt, 90 percent pass the official tests. Of the graduates who fail the pretest on the first attempt, 30 percent pass the official test. Let $A_i$ denotes that a graduate passes the pretest on the first attempt, $X$ that the graduate fails, and $B_j$ that the graduate passed the official test.

(1) (6%) Calculate the following probabilities: $P(B_j|A_i)$, $P(B_j|X)$, and $P(A_i|B_j)$

(2) (5%) A graduate has passed the official test. Obtain the posterior probabilities that this graduate did not pass the pretest.

(3) (4%) Is the pass of the official driving test independent of the pretest?

2. (15%) A Finance Professor of National Central University reported that the mean of book-to-market (BM) for firms listed in New York Stock Exchange (NYSE) is 0.82, while the mean of OTC stocks is 0.77. The sample consists of 300 NYSE stocks and 400 OTC stocks. Based on historical data, the population standard deviations for the BM can be assumed known at 0.30 for NYSE and 0.20 for OTC.

(1) (6%) Do the sample data indicate that NYSE stocks have a higher BM than OTC stocks? Use $\alpha=0.05$

(2) (4%) What is the $p$-value?

(3) (5%) What is the probability of committing a Type II error when the actual mean difference of BM between NYSE and OTC stocks is 0.0629?

3. (20%) Professor Lu in Institute of Finance, NCU, developed the following estimated regression equation relating final Financial Management score to the student’s gender, college GPA, and the number of hours spent on study per week.

$$\hat{y} = 25 + 5 \cdot X_1 + 10 \cdot X_2 + 8 \cdot X_3$$

where

$y$: final Financial Management score

$X_1$: a dummy variable set to 1 if female student and 0 otherwise...

$X_2$: college GPA

$X_3$: number of hours spent on study per week

A portion of the SAS computer output follows.

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean square</th>
<th>F Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>(A)</td>
<td>16000</td>
<td>(E)</td>
<td>(G)</td>
</tr>
<tr>
<td>Error</td>
<td>(B)</td>
<td>(C)</td>
<td>(F)</td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>63</td>
<td>(D)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$S^2 = 20$

Dependent Mean 55

R-square=

Adj R-sq=

Parameter Estimates

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter Estimate</th>
<th>Standard Error</th>
<th>t-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>?</td>
<td>15</td>
<td>?</td>
</tr>
<tr>
<td>$X_1$</td>
<td>?</td>
<td>2.0</td>
<td>(J)</td>
</tr>
<tr>
<td>$X_2$</td>
<td>?</td>
<td>8.0</td>
<td>?</td>
</tr>
<tr>
<td>$X_3$</td>
<td>?</td>
<td>2.0</td>
<td>(K)</td>
</tr>
</tbody>
</table>

(1) (7%) What are the vales of (A), (B), (C), (D), (E), (F), and (G)?

(2) (8%) What are the vales of (H), (I), (J), and (K)?

(3) (3%) Did the estimated regression equation provide a good fit to the data? Explain.

(4) (2%) Is gender a significant factor in Financial Management score?
4. 設 \( N \) 表某段期間發生火警的次數，已知隨機變數 \( A \) 之機率密度函數 (p.d.f.) 為
\[
u(x) = \begin{cases} \frac{1}{10} x^2, & x > 0 \\ 0, & \text{其他} \end{cases}
\]
若 \( N \) 之條件分頌 (Conditional Distribution) 在給定 \( A = x \) 情況下屬於普拉松分配 (Poisson Distribution) 具參數 \( \lambda \)，即 \( X | A = \lambda \sim \text{Poisson}(\lambda) \)，試求 \( P(N = n) \)。(10 分)

5. 令 \( X_1, X_2, \ldots, X_n \) 為一組來自 Gamma\((\alpha, \lambda)\) 母體之隨機樣本，其分配函數為
\[
f(x; \alpha, \lambda) = \frac{x^{\alpha-1} e^{-x/\lambda}}{\Gamma(\alpha) \lambda^\alpha}, \quad \text{若 } x > 0,
\]
其中 \( \Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t} dt \)，\( \alpha > 0 \)，試以動差法 (Method of Moment) 導出母體參數 \( \alpha \) 與 \( \lambda \) 的點估計元 (estimator)。(10 分)

6. 設 \( X_1, X_2 \) 表 p.d.f. 為
\[
f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{w} \end{cases}
\]
的分布中之組合獨立隨機樣本，試計算條件機率
\[
Pr(X_1 < X_2 \mid X_1 < 3X_2) \text{ 之值。}(10 \text{ 分})
\]

7. 設隨機變數 \( X_1, X_2 \) 為獨立且服从二項分配，其參數分別為 \( n_1, p_1 = 0.5, n_2, p_2 = 0.5 \)，試求 \( Y = X_1 - X_2 + n_2 \) 之參數。(10 分)

8. 令 \( X \) 為普拉松分配 (Poisson Distribution) Poisson(\( \lambda \)) 的隨機變數，請定義並計算此分配之偏度 (Skewness) 與峰度 (Kurtosis)。(10 分)（提示：用動差生成函數 (Moment Generating Function))