1. Evaluate
(a) \( \lim_{x \to 0} \frac{1}{\frac{1}{x} + 3x} - \frac{1}{x} \)

(b) \( \lim_{x \to 0} \frac{\tan x}{x} \)

(14%) (28%)

2. Find the derivatives \( f(x) \) of the following functions.
(a) \( f(x) = \frac{x}{\sin x} \)  
(b) \( f(x) = \tan x - x \ln x \)
(c) \( f(x) = \ln(\sin x) \)  
(d) \( f(x) = 2 \ln x \)

(28%)

3. Find the following integrals.
(a) \( \int (\sec 6x)^2 \, dx \)  
(b) \( \int x \ln x \, dx \)
(c) \( \int \frac{4}{x^2 - 1} \, dx \)  
(d) \( \int_0^3 \sqrt{9 - x^2} \, dx \)

(28%)

4. Find all points on the curve with equation \( x^2 - xy + y^2 = 3 \) at which the tangent line is horizontal.

(10%)

5. Test for convergence for the following series.
(a) \( \sum_{n=0}^{\infty} \frac{2^n}{n!} \)  
(b) \( \sum_{n=2}^{\infty} \frac{\log n (1/n)}{n^3} \)

(10%)

6. Prove that no straight line can be tangent to the graph of \( y = x^2 \) at two different points.

(10%)