# 國立中央大學102學年度碩士班考試入學試題卷

所別：財務金融學系碩士班 乙組(一般生)　科目：微積分　共 1 頁 第 1 頁

本科考試禁用計算器

＊請在試卷答案卷（卡）內作答

參考用

(20%) 1. Find the volume of the region enclosed by the surfaces $z = x^2+3y^2$ and $z = 8-x^2-y^2$.

(20%) 2. When we try to fit a line $y = mx + b$ to a set of numerical data points $(x_1, y_1)$, $(x_2, y_2)$,..., $(x_n, y_n)$, we usually choose the line that minimize the sum of the squares of the vertical distances from the points to the line. In theory, this means finding the values of $m$ and $b$ that minimize the value of the function

$$f(m, b) = (mx_1 + b - y_1)^2 + (mx_2 + b - y_2)^2 + ... + (mx_n + b - y_n)^2.$$

Please show what are the optimal values of $m$ and $b$ (expressed by $x_1, x_2, ..., x_n$ and $y_1, y_2, ..., y_n$).

(20%) 3. Use Taylor's formula to find a quadratic approximation of $f(x, y) = \cos(x)\cos(y)$ at the origin. Estimate the maximum error in the estimation if $|x| \leq 0.1$ and $|y| \leq 0.1$.

(20%) 4. Please compute $\frac{\partial C(S,r)}{\partial S}$ and $\frac{\partial C(S,r)}{\partial r}$, where $C(S, r)$ is defined as follows:

$$C(S, r) = S\Phi(d_1) - Ke^{-rT}\Phi(d_2),$$

where $d_1 = \frac{\left(\ln\left(\frac{S}{K}\right)+\left(r+\frac{1}{2}\sigma^2\right)T\right)}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma\sqrt{T}$, $\Phi(a) = \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$, and $K$, $T$ and $\sigma$ are constant .

(20%) 5. If $X_1$ and $X_2$ is a random sample form a standard normal distribution, find the joint p.d.f. of $Y_1 = X_1^2 + X_2^2$ and $Y_2 = X_2$ and the marginal p.d.f. of $Y_1$. Hint: Note that the sapce of $Y_1$ and $Y_2$ is given by $-\sqrt{y_1} < y_2 < \sqrt{y_1}$, $0 < y_1 < \infty$.