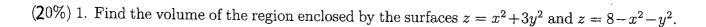
所別:財務金融學系碩士班 乙組(一般生) 科目:微積分 共 1 頁 第 1 頁

本科考試禁用計算器

-^ \*請在試卷答案卷(卡)內作答



(20%) 2. When we try to fit a line y = mx + b to a set of numerical data points  $(x_1, y_1)$ ,  $(x_2, y_2), ..., (x_n, y_n)$ , we usually choose the line that minimize the sum of the squares of the vertical distances from the points to the line. In theory, this means finding the values of m and b that minimize the value of the function

$$f(m,b) = (mx_1 + b - y_1)^2 + (mx_2 + b - y_2)^2 + \dots + (mx_n + b - y_n)^2.$$

Please show what are the optimal values of m and b (expressed by  $x_1, x_2, ..., x_n$  and  $y_1, y_2, ..., y_n$ ).

(20%) 3. Use Taylor's formula to find a quadratic approximation of  $f(x,y) = \cos(x)\cos(y)$  at the origin. Estimate the maximum error in the estimation if  $|x| \le 0.1$  and  $|y| \le 0.1$ .

(20%) 4. Please compute  $\frac{\partial C(S,r)}{\partial S}$  and  $\frac{\partial C(S,r)}{\partial r}$ , where C(S,r) is defined as follows:

$$C(S,r) = S\Phi(d_1) - Ke^{-rT}\Phi(d_2),$$

where  $d_1 = \frac{\left(\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T\right)}{\sigma\sqrt{T}}$ ,  $d_2 = d_1 - \sigma\sqrt{T}$ ,  $\Phi(a) = \int_{-\infty}^a \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$ , and K, T and  $\sigma$  are constant.

(20%) 5. If  $X_1$  and  $X_2$  is a random sample form a standard normal distribution, find the joint p.d.f. of  $Y_1 = X_1^2 + X_2^2$  and  $Y_2 = X_2$  and the marginal p.d.f. of  $Y_1$ . Hint: Note that the sapce of  $Y_1$  and  $Y_2$  is given by  $-\sqrt{y_1} < y_2 < \sqrt{y_1}$ ,  $0 < y_1 < \infty$ .

