1. (10%) Let f satisfy
$$f'(x) = 6 \cdot (K_0 - f(x)), f(0) = 2,$$
where K_0 is a positive constant.

   (a) Find the equation of the tangent line of the graph f at the point $A(0, 2)$.

   (b) Prove : $\lim_{x \to \infty} f(x) = K_0$.

2. (20%) (a) Determine whether the following series converges or diverges. Why ?

   (i) $\displaystyle\sum_{k=2}^{\infty} \frac{\ln k}{k^{1.5}}$ (ii) $\displaystyle\sum_{n=1}^{\infty} \left(n^{\frac{1}{n}} - 1\right)^n$

   (b) Find the intervals of convergence of the following power series respectively :

   (i) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3} x^n$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!} x^n$

3. (10%) A company has determined that its total revenue (in dollars) for a product can be modeled by
$$R = -x^3 + 450x^2 + 52500x,$$
where x is the number of units produced (and sold). What production level will yield a maximum revenue ?

4. (20%) Evaluate the following integrals.

   (a) $\int_0^1 x^3 e^x dx$ (b) $\int x^3 (\ln x) dx$

   (c) $\int \frac{x^5 + x + 1}{x^2 - 1} dx$ (d) $\int_{\ln 2}^{\ln 3} \frac{15}{(e^x + e^{-x})^2} dx$

5. (20%) A manufacturer's production is modeled by the Cobb-Douglas function
$$f(x, y) = 100 x^{\frac{3}{4}} y^{\frac{1}{4}},$$
where x represents the units of labor and y represents the units of capital. Each labor unit costs \$150 and each capital unit costs \$250. The total expenses for labor and capital cannot exceed \$50,000. Find the maximum production level.

6. (20%)

   (a) If $f(x, y) = x \ln \frac{y^2}{x}$, find the value of $\frac{\partial f}{\partial x}(4, 2) + \frac{\partial f}{\partial y}(4, 1)$.

   (b) Let $f(x, y) = \frac{xy}{x^2 + y^2}$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Show that $D_1 f(0, 0)$ and $D_2 f(0, 0)$ exist, but f is not continuous at $(0, 0)$.