Instructions: Answer the following questions. Make and state your own assumptions for questions where the information is not sufficient for you to solve them. For example, if you need the corresponding p-value of a normally distributed random variable evaluated at 2.5, you may indicate the value as, for example, $Pr(x \geq 2.5)$, where $x \sim \mathcal{N}(0,1)$.

1. (10 points) Suppose $x_1, ..., x_{n1} \overset{iid}{\sim} \mathcal{N}(\mu_1, \sigma_1^2)$ and $y_1, ..., y_{n2} \overset{iid}{\sim} \mathcal{N}(\mu_2, \sigma_2^2)$ are two independent random samples. Suppose also that $n_1 = n_2$. And we know that the sample estimates for these parameters are: $\bar{x}_1, \bar{x}_2, s_1^2, s_2^2$. A statistician suspects that the variance of the first sample is twice the variance of the second sample. So he specifies his null and alternative hypotheses as the following: $H_0 : s_1^2 = 2s_2^2$ and $H_1 : s_1^2 \neq 2s_2^2$. Also, he wants to test if the sample means are the same for both samples, i.e., $H_0 : \bar{x}_1 = \bar{x}_2$ and $H_1 : \bar{x}_1 \neq \bar{x}_2$.
   - (a) Are there any problems with his statement?
   - (b) How will you test these two hypotheses? Please specify the statistics and their distributions under the null. Remember to specify the degrees of freedom, if any.

2. (10 points) Let Y, X_1 and X_2 denote the amount of hay in units of 100 pounds per acre, the spring rainfall in inches, and the accumulated temperature above 45 degrees in the spring, respectively. Data accumulated over several years in England yielded the following sample values: $\bar{Y} = 28, \bar{X}_1 = 4.9, \bar{X}_2 = 590. \sum x_1 y/n = 3.87, \sum x_2 y/n = -150, \sum x_1 x_2/n = -52, \sum x_1^2/n = 1.2, \sum x_2^2/n = 7,220$. Use these results to find the equation of the least squares linear regression function of Y on X_1 and X_2. Here a lower case letter denotes a variable measured from its empirical mean. Thus, $x_1 = X_1 - \bar{X}_1$, etc.

3. (10 points) Given a sample of size n, is the $(1-\alpha) \times 100\%$ confidence interval for a parameter always the same as the corresponding critical region for that parameter at α significance level? Briefly explain your answer.

4. (10 points) Suppose that on the basis of a random sample of size 250, you are to verify the claim that a population proportion is different from .30.
   - (a) If you set the rejection region to be $|\hat{p} - .30| \geq .06$, what is the size of the type I error?
   - (b) Determine the numerical value of c so that the test based on the rejection region: $|\hat{p} - .30| \geq c$ has $\alpha = .10$.

5. (10 points) Suppose in a certain statistics course, 70% of the students who turn in the assignment on time pass the mid-term examination, while 90% of the students who turn in assignment late pass the exam (strange, isn't it?). Suppose also 50% of the students turn in the homework, 50% of whom turn in the homework on time. Only 40% of those who do not turn in the homework pass the midterm. Now, a student who failed in the exam comes to your office asking for changing grade. How would you answer the following questions:
   - (a) What is the probability that the student turned in the homework on time? What is the probability that he or she did not turn in the homework?
   - (b) Calculate the percentage of the students who turn in the assignment late and pass the exam.

6. (10 points) Suppose now an economist is interested in estimating the demand for a stock, and estimates the following regression:

$$\log q_t = \beta_1 + \beta_2 \log P_t + \varepsilon_t,$$

   where $t = 1, ..., T-1$. The estimates are: $\hat{\beta}_1 = 0.5$ and $\hat{\beta}_2 = -0.7$.
   - (a) How will you predict $\log q_T$? Is $\widehat{\log q_T}$ unbiased? What is $\hat{q}_T$, is $\hat{q}_T$ unbiased?
   - (b) What is the price elasticity in this case?

7. (10 points) Suppose you are asked to estimate the following model:

$$y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + \epsilon_t,$$

   where $t = 1, ..., 250$. And you get the following results:

variable	coefficient		std. err	t-stat	p-value
β_1	-2.474		0.4667	-5.303	0.000
β_2	0.098		0.0069	14.169	0.000
β_3	-0.077		0.0322	-2.389	0.017
β_4	29.286		7.0797	4.137	0.000
R-square	0.69	mean of dependent var.	5.466		
Adjusted R-squared	A	S.D. of dependent var.	2.46		
S. E. of regression	B	Sum of squared resid.	C		
F-stat	183.587				

   - (a) What are A, B, and C ?
   - (b) What are the degrees of freedom for the F-statistic? What is the underlying null hypothesis? Check if the number 183.587 is correct.

8. (10 points) In a simple regression model, $y_t = a + bx_t + e_t$, where $t \in [1, T]$, and $e_t \sim \mathcal{N}(0, \sigma^2)$, it can be shown that the sampling distribution for the least squares estimate of b, $\hat{b}$ is normal with mean b and variance:

$$Var(\hat{b}) = \frac{\hat{\sigma}^2}{\sum(x_t - \bar{x})^2}, \tag{1}$$

   where $\hat{\sigma}^2$ is the sample variance of σ^2. Please verify or prove if the formula for the variance is correct.

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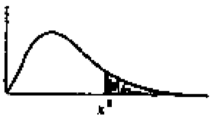
9. (10 points) Fit a Poisson density to the following data on the number of deaths that resulted per year in an army corps for 10 Prussian cavalry army corps over a period of 20 years from being kicked by a horse. The total number of sampling units here is 200. Apply the chi-square test to see whether the Poisson assumption is substantiated. Take $\alpha = 0.05\%$.

Number of deaths during the year	0	1	2	3	4
Observed frequency	109	65	22	3	1

10. (10 points) Show that the variable t^2 with ν degrees of freedom is a special case of the F variable with $\nu_1 = 1$ and $\nu_2 = \nu$. Here "t" refers to a Student's t variate.

Table V **The χ^2 distribution**

The first column lists the number of degrees of freedom (ν). The headings of the other columns give probabilities (P) for χ^2 to exceed the entry value. For $\nu > 100$, treat $\sqrt{2\chi^2} - \sqrt{2\nu - 1}$ as a standard normal variable.



ν \ P	0.995	0.975	0.050	0.025	0.010	0.005
1	$0.0^{4}3927$	$0.0^{3}9821$	3.84146	5.02389	6.63490	7.87944
2	0.010025	0.050636	5.99147	7.37776	9.21034	10.5966
3	0.071721	0.215795	7.81473	9.34840	11.3449	12.8381
4	0.206990	0.484419	9.48773	11.1433	13.2767	14.8602
5	0.411740	0.831211	11.0705	12.8325	15.0863	16.7496
6	0.675727	1.237347	12.5916	14.4494	16.8119	18.5476
7	0.989265	1.68987	14.0671	16.0128	18.4753	20.2777
8	1.344419	2.17973	15.5073	17.5346	20.0902	21.9550
9	1.734926	2.70039	16.9190	19.0228	21.6660	23.5893

Table A2 **Poisson Probabilities**

For a given value of λ, entry indicates the probability of obtaining a specified value of X

X \ λ	5.0	10	15
0	.0067	.0000	.0000
1	.0337	.0005	.0000
2	.0842	.0023	.0000
3	.1404	.0076	.0002
4	.1755	.0189	.0006
5	.1755	.0378	.0019
6	.1462	.0631	.0048
7	.1044	.0901	.0104
8	.0653	.1126	.0194
9	.0363	.1251	.0324

Table A3 **The Standardized Normal Distribution**

The entries in this table are the probabilities that a standard normal random variable is between 0 and z (the shaded area).

	Second Decimal Place in z									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4974
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981