國立中央大學八十三學年度研究所碩士班入學試題卷

系所別： 產業經濟研究所 甲 組 科目： 甲統計學 共 2 頁 第 1 頁

1. A box has 4 balls, labelled from 1 to 4, balls 1 and 2 are red, balls 3 and 4 are green. Two balls are selected at random without replacement.
(20%)

a. Are the events: (2nd ball has odd label) and (2nd ball is red) independent? Why?

b. Find the probability that at least one of the two balls is red.

2. Cars travelling North on a highway cross a given intersection at a rate of 20 per minute. Assume that the actual number of cars per minute follow a Poisson distribution.
(30%)

a. Use Chebycheff inequality to compute that probability that in a given minute there will be more than 60 cars.

Next assume that 30% of the cars passing by are made in Japan, and that cars travel independently of each other (i.e. the event that next car will be made in Japan is independent from the actual maker of any of the previous cars).

b. If in a given minute 50 cars cross the intersection, let X = the number of Japanese cars. find the distribution of X and its variance.

c. Somebody stands at the intersection, find the expected number of cars travelling North that he will watch up to and including the 5-th Japanese car.

參考用

3. $X_1, X_2, \cdots, X_n$ is a random sample and X_1 has a density of the form
(30%)

$$g(x_1|\theta) = \theta^2 x_1 e^{-\theta x_1}, \quad x_1 \geq 0, \quad \theta > 0$$
$$= 0, \quad \text{elsewhere}$$

a. Find the maximum likelihood estimator for θ.

b. Find the Cramér-Rao bound for the variance of unbiased estimators of $\lambda(\theta) = \theta^2$.

c. Find the method of moments estimator of θ.

4. A single observation X is distributed uniformly on the interval $[0, \theta]$, $\theta > 0$.
(20%) Calculate the risk function for the decision function $d(x) = cx^2$ when the loss function is quadratic, $L(\theta, a) = (\theta - a)^2$.