

# 國立中央大學九十一學年度碩士班研究生入學試題卷

所別: 資訊管理學系 丁組 科目: 離散數學 共 / 頁 第 / 頁

1. Prove that if 101 integers are selected from the set  $B = \{1, 2, 3, \dots, 200\}$ , then there are two integers such that one divides the other. (15%)
2. Prove that if  $p$  ones and  $q$  zeros are placed around a circle in an arbitrary manner, where  $p, q$ , and  $k$  are positive integers satisfying  $p \geq kq$ , the arrangement must contain at least  $k$  consecutive ones. (15%)
3. Prove that the set of primes is infinite. (10%)
4. Let  $|A|$  denote the cardinality of set  $A$ . If  $|A| = 30$  and the equivalence relation  $R$  on  $A$  partitions  $A$  into disjoint equivalence classes  $A_1, A_2$ , and  $A_3$ , where  $|A_1| = |A_2| = |A_3|$ , what is  $|R|$ ? (10%)
5. Let  $a_n = \sum_{i=0}^n i^3$ . Find and solve the recurrence relation for  $a_n$ . (15%)
6. Solve the following recurrence relation. (15%)  
$$b_{n+2} = b_{n+1}^2 / b_n, n \geq 0; b_0 = 1, b_1 = 2.$$
7. Draw a depth-first spanning tree and a breadth-first spanning tree of the complete bipartite graph  $K_{3,3}$ . (20%)