國立中央大學八十七學年度碩士班研究生入學試題卷

所別： 資訊管理研究所 丁組 科目： 離散數學 共 1 頁 第 1 頁

1. At the Gamma Kappa Phi sorority the 15 sisters who are seniors line up in a random manner for a graduation picture. Two of the sisters are Amy and Mary. What is the probability that this graduation picture will find
   (a) Mary at the center position in the line? (2%)
   (b) Mary and Amy standing next to each other? (2%)
   (c) exactly five sisters standing between Amy and Mary? (3%)
   (d) Amy standing somewhere to the left of Mary? (3%)

2. Let $A = \{x, a, b, c, d\}$.
   (a) How many closed binary operations f on A satisfy $f(a,b) = c$? (2%)
   (b) How many of the functions in part (a) has x as an identity? (2%)
   (c) How many of the functions in part (a) has an identity? (3%)
   (d) How many of the functions in part (c) are commutative? (3%)

3. A point $P(x, y)$ in the Cartesian plane is called a *lattice point* if x and y are integers. Given distinct lattice points $P_1(x_1, y_1), P_2(x_2, y_2), \ldots, P_n(x_n, y_n)$, determine the smallest value of n that guarantees the existence of $P_i(x_i, y_i)$, $P_j(x_j, y_j)$, $1 \le i < j \le n$, such that the midpoint of the line connecting $P_i(x_i, y_i)$ and $P_j(x_j, y_j)$ is also a lattice point. (10%)

4. Let A be a nonempty set and fix the set B, where $B \subseteq A$. Define the relation $\mathcal{R}$ on the power set of A by $X\mathcal{R}Y$, for $X, Y \subseteq A$, if $B \cap X = B \cap Y$.
   (a) If $A = \{1, 2, 3\}$ and $B = \{1, 2\}$, find the partition of the power set of A induced by $\mathcal{R}$. (4%)
   (b) If $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2, 3\}$, find the equivalent class of X if $X = \{1, 3, 5\}$. (3%)
   (c) For $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2, 3\}$, how many equivalent classes are in the partition induced by $\mathcal{R}$? (3%)

5. How many positive integers between 1 and 30 (inclusive) must we select in order to guarantee that we have two integers — say, x and y — in our selection whose greatest common divisor is greater than 1? (10%)

6. Solve the following recurrence relations.
   (a) $a_n + na_{n-1} = n!$ for $n \ge 1$, $a_0 = 1$. (5%)
   (b) $a_{n+2} - 4a_{n+1} + 4a_n = 0$ for $n \ge 0$, $a_0 = 1$, $a_1 = 4$. (5%)
   (c) $a_{n+2} - 5a_{n+1} + 6a_n = 4n$ for $n \ge 0$, $a_0 = 5$, $a_1 = 10$. (5%)
   (d) $a_{n+2} - 8a_{n+1} + 15a_n = 6 \cdot 3^n + 10 \cdot 5^n$ for $n \ge 0$, $a_0 = 2$, $a_1 = 10$. (5%)

7. Prove that the complements of two isomorphic simple graphs are also isomorphic. (10%)

8. The Wimbledon tennis championship is a single-elimination tournament wherein the winner of a match advances to the next round and the loser is eliminated. If 32 players compete in the singles championship and all players start from the first round. Calculate the following numbers in this particular championship.
   (a) The number of matches a player must play to win the championship. (5%)
   (b) The number of total matches that must be played to determine the champion. (5%)
   (c) The number of different arrangements of players in the first round. (5%)
   (d) The number of initial arrangements so that palyer 1 and player 2 have a chance to meet in the final match. (5%)