

所別：企業管理學系碩士班 一般類組(甲組) 科目：微積分

共 10 大題，每大題 10 分，共 100 分。須詳細寫出演算過程，否則不予計分。

1. Find the following limits.

(a) $\lim_{x \rightarrow (\pi/4)^-} (\tan x)^{\tan(2x)}$. (b) $\lim_{x \rightarrow 0^+} x \int_x^1 \frac{\cos t}{t^2} dt$.

2. (a) Find $f'(0)$ if $f(x) = \begin{cases} e^{-1/x^2}, & x \neq 0, \\ 0, & x = 0. \end{cases}$

(b) Suppose that the function f is differentiable at $x = 1$ and $\lim_{h \rightarrow 0} \frac{f(1+h)}{h} = 5$. Find $f(1)$ and $f'(1)$.

3. Find the point on the graph of $z = x^2 + y^2 + 10$ nearest the plane $x + 2y - z = 0$.

4. Let

$$f(x, y) = \begin{cases} \frac{xy(x^2 - y^2)}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0), \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$$

Find $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$.

5. Test the series for convergence or divergence using any appropriate test. Identify the test used and explain your reasoning.

(a) $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n - \ln n}$. (b) $\sum a_n$, where $a_n = \begin{cases} n/2^n, & n \text{ odd} \\ 1/2^n, & n \text{ even.} \end{cases}$

6. Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point $(2, 0)$ in the direction of $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$.

7. Evaluate the following integrals.

(a) $\int_0^1 \int_{y/2}^{2x} e^{-x^2} dy dx$. (b) $\int_0^1 \int_0^{1-x} \sqrt{x+y}(y-2x)^2 dy dx$.

8. Find the work done by $\mathbf{F}(t) = (y - x^2)\mathbf{i} + (z - y^2)\mathbf{j} + (x - z^2)\mathbf{k}$ over the curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$, $0 \leq t \leq 1$, from $(0, 0, 0)$ to $(1, 1, 1)$.

9. Let $f(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(x+1)^k}{3k}$. Find the interval of convergence of $f(x)$ and $f'(x)$.

10. The Cobb-Douglas production function for a new product is given by

$$N(x, y) = 16x^{0.25}y^{0.75}$$

where x is the number of units of labor and y is the number of units of capital required to produce $N(x, y)$ units of the product. Each unit of labor costs \$50 and each unit of capital costs \$100. If \$500,000 has been budgeted for the production of this product, how should this amount be allocated between labor and capital in order to maximize production?