所別：**企業管理學系碩士班**　　　　　**甲組** 科目：**微積分**

1. (10%) Let $a \in R$ and define the sequence $a_1, a_2, \ldots$ in R by $a_1 = a$, and $a_n = a_{n-1}^2 - a_{n-1} + 1$ if $n > 1$. For what $a \in R$ is the sequence $\{a_n\}$ convergent ? Compute the limit in the cases of convergence.

2. (10%) Let $F(x) = \int_0^x \frac{e^{3t}}{1+t^4}\, dt$, and $f(x) = F(x^2 + x)$. Find $f'(x)$.

3. (15%) Find the set of all x for which the given series converges.
   (a) $\sum_{n=1}^{\infty} \frac{n! x^{2n}}{2^n}$
   (b) $\sum_{n=1}^{\infty} \frac{1}{n^2} \left(\frac{2x}{1+x} \right)^n$
   (c) $\sum_{n=1}^{\infty} \frac{x^n}{n^{1+\frac{1}{n}}}$

4. (15%) Find maximum and minimum points and values for $f(x) = -3x^5 + 5x^3$ on $[-1.2, 1.2]$.

5. (15%) Find each integral.
   (a) $\int_{-1}^1 \sqrt{6 + |x|}\, dx$
   (b) $\int_0^1 e^{\sqrt{x}}\, dx$
   (c) $\iint_D e^{(x^2 + y^2)} dx dy$, where $D = \{(x,y) | x^2 + y^2 \leq 1\}$.

6. (15%) Find the local maxima and minima for $f(x, y) = (x^2 + 3y^2)e^{1-x^2-y^2}$.

7. (20%) The production function for a company is

$$f(x, y) = 100x^{0.25} y^{0.75}$$

where x is the number of units of labor and y is the number of units of capital. Suppose that labor costs \$48 per unit and capital costs \$36 per unit. The total cost of labor and capital is limited to \$100,000.
(a) Find the maximum production level for this manufacturer.
(b) Find the marginal productivity of money.

參考用