國立中央大學九十一學年度碩士班研究生入學試題卷

所別: 企業管理學系 甲戊組 科目: 統計學 共 1 頁 第 1 頁

1. Assume the simple linear regression model is given as

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i, i = 1, 2 \cdots, n$$

Prove the Sum of squares about mean = Sum of square due to regression + Sum of square about error; i.e.,

$$\sum_{i=1}^{n} (Y_i - \bar{Y})^2 = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 + \sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2$$

$\hat{Y}_i$ is the fitted value by least square method.

(15%)

2. Each customer who enters Mova's store will purchase a laptop PC with probability p. If the number of customers entering the store is Poisson distributed with mean λ, what is the probability that Mova does not sell any laptop PC? (15%)
3. Conduct a linear regression model for analyzing a designed two-way experiment, which one factor is with two levels and another is with three levels. Describe how you would test for interaction (Include the hypothesis, test statistics, and rejection region). (15%)
4. Show that the χ^2 test statistics for a 2x2 contingency table is equal to Z^2, where Z is the test statistic for the two-sample proportion H_0 $p_1 = p_2$ (15%)
5. . If the paired random sample with correlation $\rho = corr(X, Y)$ is given as $\{X_i, Y_i\}, i = 1,2 \cdots, n$, derive the variance of difference mean $\bar{D}$, $D_i = X_i - Y_i, i = 1,2, \cdots, n$ in terms of ρ. (15%)
6. The distribution of the amount of milk (in tons) sold by a local dairy in a given week is a continuous random variable X with probability density function

$$f(x) = \begin{cases} 1.5(1-x^2) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$ (15%)

Compute the median for X.

7. Suppose a health administrator wants to compare the unoccupied bed space for three hospitals in the same city. She randomly selects 10 different days from the records of each hospital and lists the number of unoccupied beds for each day. It is conceivable that the population distribution of data may be skewed to the right. Describe how you would test for the hypothesis that average number of unoccupied beds are equal for all three hospitals.(Include the test statistics, and rejection region). (10%)