

# 國立中央大學九十學年度碩士班研究生入學試題卷

所別: 企業管理學系 乙組 科目: 微積分 共 一 頁 第一頁

1. (10%) Find the following limits

$$(1) \lim_{x \rightarrow \infty} 3x \sin \frac{1}{x}$$

$$(2) \lim_{n \rightarrow \infty} (n^2 + n)^{\frac{1}{n}}$$

2. (10%) Determine the convergence or divergence of

$$(1) \sum_{n=0}^{\infty} \frac{(-1)^{n+1} n!}{1 \times 3 \times 5 \times \cdots \times (2n+1)}$$

$$(2) \sum_{n=1}^{\infty} \frac{4^n}{3^n - 1}$$

3. (10%) Find  $a, b, c$  and  $d$  such that  $f(x) = ax^3 + bx^2 + cx + d$  has relative maximum at  $(2, 4)$ , relative minimum at  $(4, 2)$  and inflection point at  $(3, 3)$ .

4. (20%) Let  $f(x) = \begin{cases} 0, & x = 0 \\ x \sin \frac{1}{x}, & x \neq 0 \end{cases}$  and  $g(x) = \begin{cases} 0, & x = 0 \\ x^2 \sin \frac{1}{x}, & x \neq 0 \end{cases}$ .

Show that

(1)  $f$  is continuous but not differentiable at  $x = 0$ .

(2)  $g$  is differentiable at  $0$ , and find  $g'(0)$ .

5. (20%) Compute

$$(1) \int \frac{x^2 + 5x + 2}{(x+1)(x^2+1)} dx$$

$$(2) \int_0^{\infty} e^{-4x^2} dx$$

6. (10%) Find the maximum value of

$$f(x, y) = 8xy, \quad x > 0, y > 0$$

subject to the constraint  $(x^2/3^2) + (y^2/4^2) = 1$ .

7. (10%) Let  $f(x) = \ln[2(1+x)]$ , find the Taylor series of  $f(x)$  in powers of  $x$  and determine the interval of convergence of this series.

8. (10%) Assume that  $f$  is a continuous function and that

$$\int_0^x tf(t)dt = \sin x - x \cos x.$$

Find  $f'(\pi/2)$ .

參考用