國立中央大學八十七學年度碩士班研究生入學試題卷

所別: 企業管理研究所 丙組 科目: 微積分 共 / 頁 第 / 頁

- SHOW YOUR WORK & GOOD LUCK !

1. (10%) Sketch the graph of a continuous function f that satisfies all the stated conditions.

   $f(1) = 4$;

   $f'(x) > 0$ if $x < 1$;

   $f'(x) < 0$ if $x > 1$;

   $f''(x) > 0$ for all $x \neq 1$.

2. (16%) Calculate

   (a). (8%) $\lim_{x\to 0} \frac{x\cos x + e^{-x}}{x^2}$

   (b). (8%) $\lim_{x\to 0} (1+3x)^{\frac{2}{x}}$

3. (30%) Calculate the following integrals.

   (a). (10%) $\int_0^1 x^3 e^{-x^2}\, dx$

   (b). (10%) $\int \frac{\sqrt{1-x^2}}{x^2}\, dx$

   (c). (10%) $\int_0^4 \frac{1}{(x-3)^2}\, dx$

4. (16%) Determine whether the following series converge or diverge. Justify your answers.

   (a). (8%) $\sum_{n=1}^{\infty} \frac{\sin n + 2^n}{n+5^n}$

   (b). (8%) $\sum_{n=1}^{\infty} ne^{-n^2}$

5. (16%) An object is situated in a rectangular coordinate system such that the temperature T at the point (x, y, z) is given by $T = 4x^2 - y^2 + 16z^2$.

   (a). (8%) Find the rate of change of T at the point $P(4, -2, 1)$ in the direction of the vector $\mathbf{a} = 2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$.

   (b). (8%) In what direction does T increase most rapidly at P ? What is this maximum rate of change of T at P ?

6. (12%) Sketch the region bounded by the graphs of the given equations $y = x^2$, $y + z = 4$ and $z = 0$ and use a triple integral to find its volume.