

國立中央大學103學年度碩士班考試入學試題卷

所別：企業管理學系碩士班 一般甲組(一般生) 科目：工程數學 共 1 頁 第 1 頁

本科考試禁用計算器

*請在試卷答案卷(卡)內作答

1. Solve the following homogeneous differential equation with the specified auxiliary conditions: (20%)

$$\frac{d^3 y(t)}{dt^3} + \frac{d^2 y(t)}{dt^2} - \frac{dy(t)}{dt} - y(t) = 0, \quad y(0) = 1, \quad y'(0) = 1, \quad y''(0) = -2$$

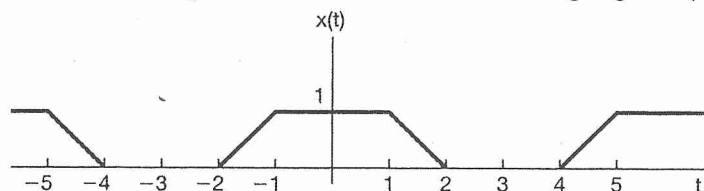
2. For the continuous-time periodic signal

$$x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right),$$

determine the fundamental frequency ω_0 and the Fourier series coefficients a_k such that

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}. \quad (20\%)$$

3. Determine the Fourier series representations for the following signal: (20%)



4. Consider the Fourier transform pair

$$e^{-|t|} \xleftrightarrow{\mathcal{F}} \frac{2}{1+\omega^2}.$$

- (a) Use the appropriate Fourier transform properties to find the Fourier transform of $te^{-|t|}$. (10%)
 (b) Use the result from part (a), along with the duality property, to determine the Fourier transform of

$$\frac{4t}{(1+t^2)^2}. \quad (10\%)$$

5. Consider the signal $x(t) = e^{-5t}u(t-1)$, and denote its Laplace transform by $X(s)$.

- (a) Evaluate $X(s)$ and specify its region of convergence. (10%)
 (b) Determine the values of the finite numbers A and t_0 such that the Laplace transform $G(s)$ of $g(t) = Ae^{-5t}u(-t-t_0)$ has the same algebraic form as $X(s)$. What is the region of convergence corresponding to $G(s)$? (10%)

參考用