所別：水文科學研究所碩士班 一般生 科目：流體力學

1. Fundamentals

a. Write down the mathematic expression of gradient, vorticity, and divergence, their corresponding integral form if appropriate, and give some examples to explain their physical meaning. (10 points)

b. Write down the continuity equation in Lagrangian form and Eulerian form, and explain its meaning. (10 points)

2. Please express each of the RHS terms in equation (1) in graphic form (i.e. to mark the appropriate area in Fig. 1 corresponding to the terms), and describe physical mean of each term. (15 points)

$$\frac{D}{Dt}\int_{z=a(t)}^{z=b(t)} T(z,t)dz = \int_{a(t)}^{b(t)} \frac{\partial T}{\partial t}dz + T(b,t)\frac{db}{dt} - T(a,t)\frac{da}{dt} \qquad (1)$$

[the Leibnitz's theorem applied to an ocean temperature profile as a function of depth and time]

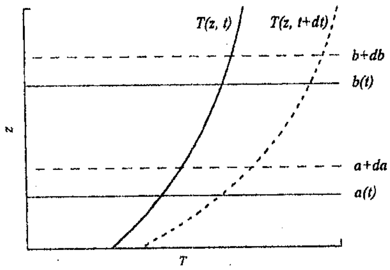

Fig. 1

3. The momentum equation in a fluid can be expresses as

$$\rho\frac{D\vec{V}}{Dt} = -\nabla p - \rho g\vec{k}, \qquad (2)$$

where $\vec{V} = (u, v, w)$ is velocity vector, ρ is density, p is pressure, g is gravity

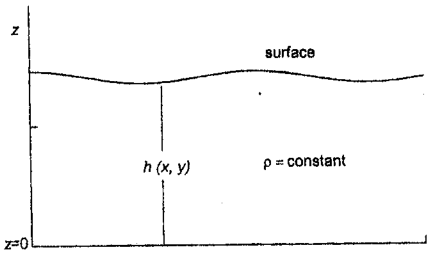

Fig. 2

a. Please explain the meaning of each term in (2). (4 points)

b. Write down the vertical component of the momentum equation, and draw a picture showing the force balance at a point in the ocean (Fig. 2). (4 points)

c. In the ocean, Dw/Dt is normaly negligibly small, so the vertical momentum equation can be simplified to a hydrostatic equation. Please write down the equation. (4 points)

d. Please integrate the hydrostatic equation from surface to a depth z to obtain

$$p = \rho g(h-z) + p_0, \qquad (3)$$

注意：背面有試題

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where $p_0 = p(x, y, h)$. (4 points)

e. Assume p_0 is negligibly small in (3), please express the term ∇p as a function of h. Is ∇p function of z ? If not, are u, v function of z (assuming u, v are initially independent of z) ? (4 points)

f. From the above arguments, write down horizontal momentum equation in scalar form (5 points)

4. Consider a rotating cylindrical tank filled with water (Fig. 3).

a. Sketch the forces on a small "parcel" of fluid extracted from the center of the liquid and explain how the forces arise. (10 points)

b. Assume the water in the tank is in solid body rotation, what is the equilibrium shape of the free surface (hint: $v_\theta = r\omega$, $a_r = -\rho v_\theta^2 / r$, and the answer from Problem 3e) (10 points)

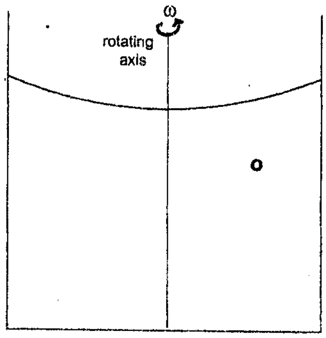

Fig. 3

c. Discuss the forces and flow after the rotation is suddenly stopped. (20 points) (hint: start from the change near the bottom where viscosity slows the motion. This will alter the force balance and result in a radial flow (which direction and why?), leading to a secondary circulation (why?). The secondary circulation spins down rotating fluid more rapidly than could mere diffusion (why?).