所別：太空科學研究所碩士班　　　科目：應用數學　共 2 頁 第 1 頁

＊請在試卷答案卷（卡）內作答

注意：作答時，請寫出計算步驟或推導或用文字說明如何獲得答案。
　　　如果只列出最後答案，卻沒有文字說明或計算步驟推導，該題將不予計分。

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1. (15 points)　Let function $y(x)$ satisfies the following ordinary differential equation.

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - x = 0$$

(a) Find the general solution of the function $y(x)$.

(b) If $y(x)$ satisfies the following initial conditions

$$y = 0, \quad \frac{dy}{dx} = \frac{5}{4}, \quad \frac{d^2y}{dx^2} = \frac{1}{4} \quad \text{at} \quad x = 0.$$

Determine the value of $y(x)$ at $x = 1$.

2. (10 points)　Let function $f(x,t)$ satisfy the following partial differential equation

$$\frac{\partial f(x,t)}{\partial x} + \frac{1}{c}\frac{\partial f(x,t)}{\partial t} = 0$$

Let $c = 2$. Table 1 shows the data points of $f(x,t)$ at the given x and t

Find the value of $f(x,t)$ at

(a) $x = 10$ and $t = 10$

(b) $x = 4$ and $t = 3$

Table 1

x	t	$f(x,t)$
5	9	222
−3	4	174
2	6	153
4	12	453
16	9	57

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3. (5 points)　Consider the following two double integrals.

$$I_1 = \int_0^2 dy \int_0^{2y} f(x,y)\, dx \quad \text{and} \quad I_2 = \int_A^B dx \int_C^D f(x,y)\, dy$$

Let $I_1 = I_2$. Find A, B, C, and D.

4. (20 points)　Evaluate the following integrals, where $i = \sqrt{-1}$

(a) $\displaystyle \int_0^{2\pi} \frac{dx}{\cos x + i7\sin x + 4}$

(b) $\displaystyle \int_{-\infty}^{\infty} \exp(ikx)\, dk \int_{-\infty}^{\infty} f(\xi) \exp(-ik\xi)\, d\xi$

注意：背面有試題

參考用

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5. (5 points)　Evaluate the determinates of the following two matrices

(a) $\begin{bmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \end{bmatrix}$　　　(b) $\begin{bmatrix} a & b & 0 & 0 \\ 0 & 0 & a & b \\ c & d & 0 & 0 \\ 0 & 0 & c & d \end{bmatrix}$

6. (20 points)

Let $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, $S = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, and $M = SAS^T$, where S^T is the transpose of S.

(a) Determine the Matrix M

(b) Find the eigen values λ_1, λ_2, λ_3 of the matrix M.

(c) Find the corresponding *normalized* eigen vectors $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$.

(d) Let $\mathbf{v} = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$, and $\mathbf{u} = M^{31}\mathbf{v}$.　Determine the column vector $\mathbf{u}$.

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7. (10 points)　Expand the following vector forms such that the vector differential operator

∇ will operate on one vector at a time.

(a) $\nabla \times (\mathbf{A} \times \mathbf{B})$　　　　(b) $\nabla \times (\mathbf{A} \times (\mathbf{B} \times \mathbf{C}))$

where the boldface font indicate that the variable is a vector.

8. (15 points)　Let us consider a function $f(x,y) = 9x^2 + 24xy + 16y^2 + 6x + 8y + 9$,

and three lines $L_1 : 3x + 4y = 14$, $L_2 : 4x - 3y = 2$, $L_3 : x - y = 0$.

The three lines L_1, L_2, and L_3 intersect at $x = y = 2$.

(a) Find the absolute value of $\left| \dfrac{df}{ds} \right|$ along L_1 at point $x = y = 2$.

(b) Find the absolute value of $\left| \dfrac{df}{ds} \right|$ along L_2 at point $x = y = 2$.

(c) Find the absolute value of $\left| \dfrac{df}{ds} \right|$ along L_3 at point $x = y = 2$.

注意：背面有試題