所別：<u>太空科學研究所碩士班 一般生</u> 科目：<u>應用數學</u>

1. Solve the following ordinary differential equations:

    (a) $x\dfrac{dy}{dx}+(1-x)y=xe^{x}$, (10%)

    (b) $\dfrac{d^{2}y}{dx^{2}}-\dfrac{4}{x}\dfrac{dy}{dx}+\dfrac{6y}{x^{2}}=\dfrac{21}{x^{6}}$ (10%)

2. Evaluate the following integrals:

    (a) $\displaystyle\int_{0}^{\pi} ln(sinx)\, dx$, (12%)

    (b) $\displaystyle\int_{-\infty}^{\infty}\dfrac{\sin x}{x}dx$ . (13%)

3. Expand the function $f(z)=\dfrac{2}{z(z-1)(z-2)}$ in a Laurent (or Taylor) series of powers of z in the following regions:

    (a) $0<|z|<1$, (5%)　　(b) $1<|z|<2$, (5%)　　(c) $|z|>2$. (5%)

4. Find the general solution of partial differential equation:
    $\dfrac{\partial^{2}z}{\partial x^{2}}-2\dfrac{\partial^{2}z}{\partial x\partial y}+\dfrac{\partial^{2}z}{\partial y^{2}}=0$ . (5%)

5. Find the solution of wave equation
    $\dfrac{\partial^{2}\phi}{\partial t^{2}}=c^{2}\dfrac{\partial^{2}\phi}{\partial x^{2}}$ .

    for which $\phi=x^{2}$ and $\dfrac{\partial\phi}{\partial t}=\cos x$ for all values of x when t = 0. (15%)

6. Evaluate the surface integral $\displaystyle\iint_{s}\vec{F}\bullet d\vec{\sigma}$, where $\vec{F}=x\hat{i}+y\hat{j}$,

    $s:x^{2}+y^{2}+z^{2}=1, z\geq 0$.(10%)

7. Expand the function $f(x)=\sin\dfrac{\pi x}{2l}$ in a Fourier series of period $2l$, over the interval $(-l, l)$ and sketch the original function and the function represented by series in the interval $(-3l, 3l)$. (10%)