國立中央大學九十一學年度碩士班研究生入學試題卷

所別: 太空科學研究所 不分組 科目: 應用數學 共 1 頁 第 1 頁

1. Solve the following ordinary differential equations:

   a) $\left(3y^2 - x^2\right)dx = 2xy\,dy$, (10%)

   b) $\begin{cases} 2\dfrac{dx}{dt} - 3x + y = 4e^t, \\ x + 2\dfrac{dy}{dt} - 3y = 0. \end{cases}$ (10%)

2. The position vector of a moving particle is

   $$\mathbf{r} = \mathbf{a}\cos\omega t + \mathbf{b}\sin\omega t,$$

   where t is time, $\mathbf{a}$,$\mathbf{b}$, are constant vectors, ω is a constant.

   a) Find the velocity $d\mathbf{r}/dt$, (5%)

   b) Show that the curve traced out lies in a plane , what is the normal vector of this plane. (8%)

   c) Show that the acceleration is directed toward the origin and is proportional to $|\mathbf{r}|$. (5%)

3. Evaluate the following integrals

   a) $\int_0^\infty e^{-x^2}dx$, (6%)

   b) $\int_0^{2\pi} \dfrac{dx}{1+\sin^2 x}$. (12%)

4.

   a) Find the Fourier series for the function

   $f(x) = x^2, \qquad -\pi \le x \le \pi$. (10%)

   b) Sketch the graphs of $f(x) = x^2$ and the fourier series in (a) over the interval $(-3\pi, 3\pi)$. (5%)

   c) Find the sum of the series $\sum_{n=1}^{\infty} \dfrac{1}{n^2}$. (5%)

5.

   a) If $f(P,V,T) = 0$, determine expressions for each of the following and express each result as function of $\dfrac{\partial f}{\partial T}$, $\dfrac{\partial f}{\partial V}$, and $\dfrac{\partial f}{\partial P}$:

   (i) $\left(\dfrac{\partial V}{\partial T}\right)_P$, (ii) $\left(\dfrac{\partial P}{\partial V}\right)_T$, (iii) $\left(\dfrac{\partial P}{\partial T}\right)_V$. (5%)

   b) Show that $\left(\dfrac{\partial P}{\partial T}\right)_V = -\left(\dfrac{\partial V}{\partial T}\right)_P \left(\dfrac{\partial P}{\partial V}\right)_T$. (4%)

6.

   a) Find the general solution of wave equation $\dfrac{\partial^2 \varphi}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial^2 \varphi}{\partial t^2}$. (7%)

   b) Find the specific solution of wave equation which satisfies the so-alled Cauchy conditions:

   $\varphi(x,0) = F(x)$, $\left.\dfrac{\partial \varphi(x,t)}{\partial t}\right|_{t=0} = G(x)$. (8%)