國立中央大學八十六學年度碩士班研究生入學試題卷

1. A sphere of radius R carries a charge density

$$\rho(r) = kr$$

where k is a constant.

(a) Find the electric field $\vec{E}$ and the electric potential Φ everywhere. (10%)

(b) Find the electric energy of this distribution. (5%)

2. An electric dipole of moment $\vec{p}$ along the line between the center of a conducting grounded sphere of radius R and the dipole is placed at a distance d ($R < d$) from the center of the sphere.

(a) Find the electric potential Φ inside and outside the sphere. (10%)

(b) Find the induced surface charge of the sphere. (5%)

(c) Determine the force and the torque between the dipole and the sphere. (10%)

3. A magnetic dipole $\vec{m}$ is placed in a magnetic field $\vec{B}$.

(a) Show that the interaction energy is given by

$$U = -\vec{m} \cdot \vec{B}$$

(10%)

(b) Show that the torque on the dipole is

$$\vec{N} = \vec{m} \times \vec{B}$$

(10%)

4. If two tiny wire loops, of areas $\vec{s}_1$ and $\vec{s}_2$, are situated a distance r apart. We treat them as two magnetic dipoles.

(a) Find their mutual inductance. (10%)

(b) If a current I_1 is flowing in loop 1, and we propose to turn on a current I_2 in loop 2. How much work must be done, against the mututal induced electromotive force, to keep the current I_1 following in loop 1. (5%)

5. For a conducting material with electric conductivity σ,

(a) Is there a current density inside the conductor when the electromagnetic wave propagates ? (5%)

(b) Derive the electromagnetic wave equations for the conductor. (10%)

(c) Are there plane-wave solutions ? (5%)

(d) What is the skin depth for the conductor ? (5%)