國立中央大學 110 學年度碩士班考試入學試題

所別： 太空科學與工程學系 碩士班 不分組(一般生)
太空科學與工程學系 碩士班 不分組(在職生)

科目： 應用數學

本科考試禁用計算器

＊請在答案卷(卡)內作答

請注意:作答時請寫出推導計算步驟或用文字說明清楚如何獲得答案。若只列出最後答案，卻沒有推導計算步驟或文字說明，則該題將不予計分。Show the details of all your works.

1.Solve the following ODEs.

(a) $xy'' - (2x-1)y' + (x-1)y = 0$. (12%)

(b) $y'' + 6y' + 10y = 30 * cos2t$, $y(0) = 0$, $y'(0) = 2$. The " $*$ " means convolution. (12%)

2.(a) Find the general solution of $y_1' - 0.25y_2 = 0, y_2' + y_1 = 0$ by solving the eigenvalues and the corresponding eigenvectors. (10%) (b) Discuss the solution by plotting some trajectories in the phase plane, including the shape and direction. (8%)

3.Find the steady-state temperature $T(x,y)$ in the thin rectangle plate $0 \leqq x \leqq 10$, $0 \leqq y \leqq 20$ by solving the Laplace equation $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$. The upper side of the rectangle is kept at $u(x)$ °C, the lower side is kept at 0 °C, and the other two sides are perfectly insulated. Perfectly insulated means no temperature change across the boundary. (20%)

4.Find the Fourier transform of $f(x) = xe^{-x}$, if $-1 < x < 0$; otherwise, $f(x) = 0$. (10%)

5. Show that

$$\int_0^\infty \frac{coswxsinw\pi}{w} dw = \begin{cases} \pi/2 & \text{if } 0 < x < \pi \\ \pi/4 & \text{if } x = \pi \\ 0 & \text{if } x > \pi \end{cases}$$

by Fourier integral of $f(x) = 1$, if $|x| < \pi$, $f(x) = 0$, if $|x| > \pi$. (8%)

6.Find all roots of $\sqrt[3]{-81i}$. (8%)

7.Evalute the integrate $\int_0^{2\pi} \frac{\sin\theta}{3+\cos\theta} d\theta$ by finding the residues. (12%)