國立中央大學109學年度碩士班考試入學試題

所別： 太空科學研究所 碩士班 不分組(一般生)　　共 1 頁 第 1 頁
太空科學研究所 碩士班 不分組(在職生)

科目： 應用數學

本科考試禁用計算器　　＊請在答案卷(卡)內作答

請注意:作答時請寫出推導計算步驟或用文字說明清楚如何獲得答案。若只列出最後答案，卻沒有推導計算步驟或文字說明，則該題將不予計分。Show the details of all your works.

1.Solve the following ODEs.

(a) $xy' - y - x^3 \sin^2(2y/x) = 0, y(1) = \pi$. (10%)

(b) $y'' + 2y' + 2y = 5u(t-3)e^t,\ y(0) = 0,\ y'(0) = 1.$ $u(t-3)$ is unit step function. (10%)

(c) $2x^2y''' + 8xy'' + 9y' = 0$. (10%)

(d) $y_1' - 8y_1 + y_2 = 0,\ y_2' - y_1 - 10y_2 = 0$. (15%)

2.Find $w(x,t)$ for the string of length π and when the initial velocity is zero and the initial deflection is $(5\sin x - 2\sin 3x)$ by solving the one-dimensional wave equation $\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$. (20%)

3.Find the Fourier transform of the given function. (10%)

$$f(x) = x,\ \text{if}\ -1 < x < 1,\ f(x) = 0,\ \text{otherwise.}$$

4.Reduce the given ODE, $\frac{1}{\sin\emptyset}\frac{d}{d\emptyset}\left(\sin\emptyset \frac{dH}{d\emptyset}\right) + kH = 0$, to the Legendre's equation by setting $\cos\emptyset = w$ and $k = n(n+1)$, where n is a constant. (15%)

5.Evalute the integrate $\int_{-\infty}^{\infty} \frac{x^2+1}{x^4+1} dx$ by finding the residues. (10%)

參考用