國立中央大學 108 學年度碩士班考試入學試題

所別： 太空科學研究所 碩士班 不分組(一般生)
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科目： 應用數學

本科考試禁用計算器

請注意:作答時請寫出推導計算步驟或用文字說明清楚如何獲得答案。若只列出最後答案，卻沒有推導計算步驟或文字說明，則該題將不予計分。Show the details of all your works.

1.Solve the following initial value problems.

(a) $yy' + xy^2 + 5x = 0,\ y(0) = -1.$ (10%)

(b) $y'' + 4y' + 8y = \delta(t-3),\ y(0) = 0,\ y'(0) = 4.$ (10%)

2.Given a matrix $\mathbf{A} = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$, (a) find the inverse $\mathbf{A}^{-1}$ (5%), and (b) determine the eigenvalues and the corresponding eigenvectors (15%).

3.Reduce the given ordinary differential equation, $\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} + k^2u = 0$, to Bessel's equation by setting $s = kr$ and find the general solution. k is a constant. (15%)

4.Suppose that $f(t)$ and $g(t)$ are piecewise continuous, bounded, and absolutely integrable on the t-axis. Show that the Fourier transform of the convolution of functions $f(t)$ and $g(t)$ is

$$F\{f(t) * g(t)\} = \sqrt{2\pi}F\{f(t)\}F\{g(t)\}.$$

Note that the convolution of functions $f(t)$ and $g(t)$ is defined by

$$f(t) * g(t) \equiv \int_{-\infty}^{+\infty} f(p)g(t-p)dp = \int_{-\infty}^{+\infty} f(t-p)g(p)dp.$$ (15%)

5.Find the temperature $T(x,t)$ in a laterally insulated bar of length π with $c^2 = 1$ by solving the one-dimensional heat equation, $\frac{\partial T}{\partial t} = c^2\frac{\partial^2 T}{\partial x^2}$, for the adiabatic boundary conditions, $T_x(0,t) = 0$ and $T_x(\pi,t) = 0$, and the initial temperature $T(x,0) = 10cos2x$. Note that $T_x = \partial T/\partial x$. (20%)

6.Integrate the complex function $f(z) = \frac{2z-1}{z^2+z-6}$ clockwise around the circle $|z-1|=3$. (10%)