

國立中央大學 107 學年度碩士班考試入學試題

所別：太空科學研究所 碩士班 不分組(一般生)

共 1 頁 第 1 頁

太空科學研究所 碩士班 不分組(在職生)

科目：應用數學

本科考試禁用計算器

*請在答案卷(卡)內作答

請注意：作答時，請寫出推導計算步驟或用文字說明如何獲得答案。如果只列出最後答案，卻沒有推導計算步驟或文字說明，該題將不予計分。

(1, 20%) Find a general solution of $y' + y = -\frac{x}{y}$ and show the steps of derivation.

(2, 20%) Find a general solution of the Euler-Cauchy equation. Show the details of your work.

$$x^2 y'' + xy' + 9y = 0, y(1) = 0, y'(1) = 2.5.$$

(3, 20%) Please derive a general solution of the Legendre's differential equation,

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0, \text{ in powers of } x \text{ with convergence range } |x| < 1.$$

(4, 20%) Let $\mathbf{M} = \begin{bmatrix} a & b & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \\ c & d & 0 & 0 \end{bmatrix}$, where $a > b > 0$ and $d > c > 0$,

(a) Find the determinate of the matrix \mathbf{M} (10%).

(b) Find the inverse matrix of \mathbf{M} (10%).

(5, 20%) Find the deflection, $u(x, t)$, for a vibrating elastic string in a model of the one-

dimensional wave equation, $\frac{\partial^2 u(x, t)}{\partial t^2} = c^2 \frac{\partial^2 u(x, t)}{\partial x^2}$, where c is constant in this case. The

string is fastened at the ends $x = -L$ and $x = L$. Two boundary conditions are given as $u(-L, t) = 0$ and $u(L, t) = 0$ for all $t \geq 0$. Two initial conditions at time $t = 0$ are given as

$u(x, 0) = f(x)$ and $\frac{\partial u(x, 0)}{\partial t} = g(x)$ for $-L \leq x \leq L$.

參考用