所別：<u>太空科學研究所碩士班 不分組(一般生)</u>　科目：<u>應用數學</u>　共 <u>2</u> 頁 第 <u>1</u> 頁
　　　<u>太空科學研究所碩士班 不分組(在職生)</u>

本科考試禁用計算器　　　　　　　　　　　　　　　＊請在試卷答案卷（卡）內作答

注意：背面有試題

注意：作答時，如果只列出最後答案，卻沒有文字繪圖說明或計算步驟，該題將不予計分。

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1. (10 points) [(a) 2 points, (b) 6 points, (c) 2 points]

Let us consider a probability distribution function $f(x)$, which satisfies

$$
\begin{aligned}
f(x) &= x & \text{if } 0 \le x \le 1 \\
&= 2-x & \text{if } 1 \le x \le 2 \\
&= 0 & \text{if } x > 2 \text{ or } x < 0
\end{aligned}
$$

(a) Find the average value (expected value) of this probability distribution.

(b) Find the variance of this probability distribution.

(c) Find the standard deviation of this probability distribution.

2. (10 points) [(a) 5 points, (b) 5 points]

Find the inverse matrices of the following matrices.　Verify your results.

(a) $A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 3 \end{bmatrix}$

(b) $B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 2 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 3 \end{bmatrix}$

3. (10 points)

$$
\begin{bmatrix} +\dfrac{\sqrt{3}}{2} & +\dfrac{\sqrt{2}}{4} & -\dfrac{\sqrt{2}}{4} \\[2mm] +\dfrac{1}{2} & -\dfrac{\sqrt{6}}{4} & +\dfrac{\sqrt{6}}{4} \\[2mm] 0 & +\dfrac{\sqrt{2}}{2} & +\dfrac{\sqrt{2}}{2} \end{bmatrix}
\begin{bmatrix} x \\ y \\ z \end{bmatrix}
=
\begin{bmatrix} 4 \\ -2\sqrt{3} \\ 1 \end{bmatrix}
\quad \text{Find} \quad
\begin{bmatrix} x \\ y \\ z \end{bmatrix} = ?
$$

4. (10 points)

Evaluate the following definite integral,

$$
I = \int_0^{2\pi} \frac{dx}{7 - 3\cos x + 9i \sin x}
$$

注意：背面有試題

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5. (20 points) [(a) 10 points, (b) 10 points]

Evaluate the following definite integrals, where μ and σ are positive real numbers

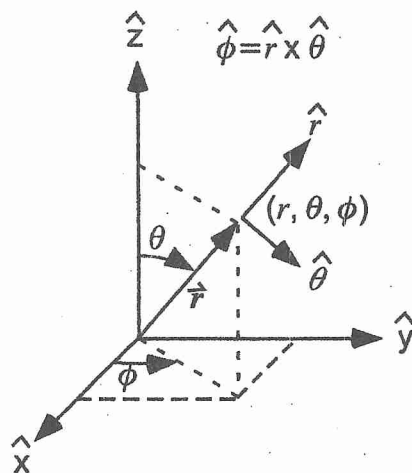(a) $I_1 = \int_{-\infty}^{+\infty} \frac{x^2}{\sqrt{2\pi}\sigma} \exp[-\frac{(x-\mu)^2}{2\sigma^2}] dx$

(b) $I_2 = \int_{-\infty}^{+\infty} \frac{(x-\mu)^4}{\sqrt{2\pi}\sigma} \exp[-\frac{(x-\mu)^2}{2\sigma^2}] dx$

6. (20 points) [(a) 5 points, (b) 5 points, (c) 10 points]

Let us consider a spherical coordinate system (r, θ, ϕ) where r is the radial distance from the origin; θ is the polar angle between the position vector $\mathbf{r} = \hat{r}r$ and the z-axis; ϕ is the azimuthal angle of the position vector $\mathbf{r}$ with respect to the x-z plane. The unit vectors $\hat{r}, \hat{\theta}, \hat{\phi}$ are parallel to the ∇r, $\nabla\theta$, and $\nabla\phi$ directions, respectively.

Let $\mathbf{A} = \hat{\phi}A_\phi = \hat{\phi}\frac{\sin\theta}{r^2}$.

(a) Determine $\nabla \cdot \mathbf{A} = ?$

(b) Determine $\nabla \times \mathbf{A} = ?$

(c) Determine $\mathbf{A} \cdot \nabla \mathbf{A} = ?$

7. (20 points) [(a) 10 points, (b) 10 points]

Given matrix $M = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

and column vector $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(a) Find the eigen values and eigen vectors of the matrix M

(b) Let $\mathbf{u} = M^{101}\mathbf{v}$. Determine the column vector $\mathbf{u}$.

注意：背面有試題