

國立中央大學102學年度碩士班考試入學試題卷

所別：太空科學研究所碩士班 不分組(一般生) 科目：電磁學 共    /    頁 第    /    頁  
太空科學研究所碩士班 不分組(在職生)

本科考試禁用計算器

\*請在試卷答案卷(卡)內作答

參考用

1. Given the electric potential  $\Phi(r, \theta, \phi) = -Ar \cos\theta + \frac{B}{4\pi\epsilon_0} \left( \frac{1}{r} - \frac{1}{R} \right) + \frac{AR^3}{r^2} \cos\theta$  in spherical coordinates  $(r, \theta, \phi)$  with constants  $A, B, R$ , and the permittivity of free space  $\epsilon_0$ ,
  - (a) Determine the electric field (5%)
  - (b) Sketch the electric field lines. (5%)
  - (c) Find the charge distribution that given rise to the potential. (5%)
  - (d) Find the electric interaction energy of the system. (5%)
  
2. A point charge  $q$  is placed at a point  $P$  in medium 1, which is at a distance  $d$  from the boundary of two semi-infinite homogeneous linear dielectrics of permittivities  $\epsilon_1$  and  $\epsilon_2$ .
  - (a) What are the boundary conditions for the potential and the electric field? (5%)
  - (b) Find the image charges. (5%)
  - (c) Find the potential in region 1 of permittivity  $\epsilon_1$ . (5%)
  - (d) Find the polarization in region 2 of permittivity  $\epsilon_2$ . (5%)
  
3. A small current circular coil of radius  $R$  lies in x-y plane with its center at the origin. It carries a stationary current  $I$  running counterclockwise as viewed from the positive z-axis. There is an external uniform magnetic field  $\vec{B} = \frac{B_0}{\sqrt{2}}(\hat{x} + \hat{y})$  that is not generated by the coil of the current  $I$ .  
 Where  $B_0$  is a constant and  $\hat{x}, \hat{y}$  are unit vectors along the x-axis and y-axis, respectively.
  - (a) Find the magnetic field at points along z-axis. (5%)
  - (b) Find the magnetic moment of the coil and express the result in (a) in terms of the magnetic moment. (5%)
  - (c) Find the force acting on the coil. (5%)
  - (d) Find the torque acting on the coil. (5%)
  
4. A uniformly magnetized sphere of radius  $R$  with the magnetization  $\vec{M} = M_0\hat{z}$  in z direction.
  - (a) Find the vector potential of the magnetic field outside the sphere. (5%)
  - (b) Find the magnetic field outside the sphere. (5%)
  - (c) Find the current distribution of a conducting sphere of radius  $R$  that produces the identical magnetic field in (b). (5%)
  
5. A rectangular wire of width  $w$  and length  $l$  in x-y plane is moving parallel to its width at a uniform velocity  $\vec{v}$  in y direction. An infinite long wire, carrying a stationary current  $I$  in x direction, coplanar with the wire, and parallel to its length is at a distance  $d$  from the nearest long side.
  - (a) Determine the electromotive force induced in the rectangular wire. (5%)
  - (b) Determine the mutual inductance of the system. (5%)
  - (c) If the current carried in the infinite long wire is time dependent  $I(t)$ , determine the electromotive force induced in the rectangular wire. (5%)
  
6. For the electromagnetic waves confined to the interior of a hollow pipe (or wave guide),
  - (a) Give the reasons for the fields inside the material of a hollow pipe are  $\vec{E} = 0$  and  $\vec{B} = 0$ . (5%)
  - (b) Find the boundary conditions for the fields at the inner wall of a hollow pipe. (5%)

