本科考試禁用計算器　　　　　　　　　　　　　　　　　　　　　　＊請在試卷答案卷（卡）內作答

注意：作答時，如果只列出最後答案，卻沒有文字繪圖說明或計算步驟，該題將不予計分。

1. (10 points)　[(a) 2 points, (b) 4 points, (c) 4 points]

Let us consider a probability distribution function, which is confined in the region between 0 and 1 with a uniform distribution profile. i.e.,

$$f(x) = 1 \quad \text{if } 0 \leq x \leq 1$$
$$= 0 \quad \text{if } x > 1 \text{ or } x < 0$$

(a) Find the average value (expected value) of this probability distribution.

(b) Find the variance of this probability distribution.

(c) Find the standard deviation of this probability distribution.

2. (10 points)

Let functions $v_x(t)$ and $v_y(t)$ satisfy the following ordinary differential equations:

$$\frac{dv_x(t)}{dt} = \omega v_y(t)$$

$$\frac{dv_y(t)}{dt} = \omega[v_0 - v_x(t)]$$

where ω and v_0 are positive constant. Please find the solution of the functions $v_x(t)$ and $v_y(t)$ with initial conditions: $v_x(t=0) = 2v_0$ and $v_y(t=0) = 0$.

3. (15 points) [(a) 5 points, (b) 5 points, (c) 5 points]

Find the inverse matrices of the following matrices. Please verify your results.

(a) $A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 4 \end{bmatrix}$

(b) $B = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 2 & 4 & 0 & 0 \end{bmatrix}$

(b) $C = \begin{bmatrix} \sin\beta & \cos\beta & 0 \\ \cos\alpha\cos\beta & -\cos\alpha\sin\beta & \sin\alpha \\ -\sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{bmatrix}$

4. (30 points)

[(a) 5 points, (b) 5 points, (c) 5 points, (d) 5 points, (e) 5 points, (f) 5 points]

Evaluate the following definite integrals, where μ, and σ are positive real numbers, and $z_0 = 5 + 5i$ and $i = \sqrt{-1}$

(a) $I_1 = \int_{-\infty}^{+\infty} \frac{1}{\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] dx$

(b) $I_2 = \int_{-\infty}^{+\infty} \frac{x^2}{\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] dx$

(c) $I_3 = \int_{-\infty}^{+\infty} \frac{z + z_0}{z - z_0} e^{iz} dz$

(d) $I_4 = \int_{-\infty}^{+\infty} \frac{z + z_0}{z - z_0} e^{-iz} dz$

(e) $I_5 = \int_{-\infty}^{+\infty} dk \int_{-\infty}^{+\infty} \cos(\xi) e^{ik(x-\xi)} d\xi$

(f) $I_6 = \int_0^{2\pi} \frac{dx}{7\cos x + i\sin x + 8}$

注意：背面有試題

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5. (10 points)

Let function $f(x,t)$ satisfy the following partial differential equation

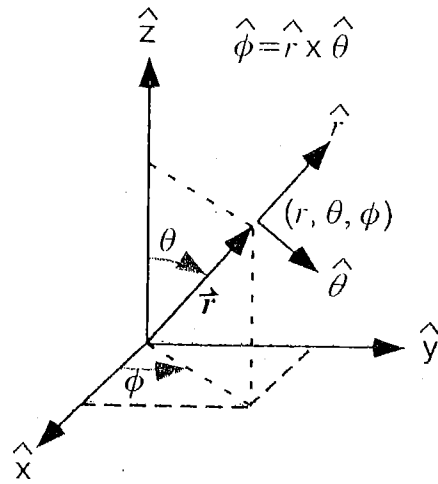$$\frac{\partial f(x,t)}{\partial x} - \frac{1}{c}\frac{\partial f(x,t)}{\partial t} = 0$$

(a) Find the general solution of $f(x,t)$

(b) Let $c = 2\,\mathrm{m/sec}$. Table 1 shows the data points of $f(x,t)$ at given x and t.

　Find the value of $f(x,t)$ at $x = 10\,\mathrm{m}$ and $t = 6\,\mathrm{sec}$

Table 1

x(m)	t(sec)	$f(x,t)$
5	4	33
10	3	36
15	2	39
20	1	42
25	0	45

6. (15 points)　[(a) 5 points, (b) 5 points, (c) 5 points]

Let us consider a spherical coordinate system (r, θ, ϕ), where r is the radial distance from the origin; θ is the polar angle between the position vector $\mathbf{r} = \hat{r}r$ and the z-axis; ϕ is the azimuthal angle of the position vector $\mathbf{r}$ with respect to the x-z plane. The unit vectors $\hat{r}, \hat{\theta}, \hat{\phi}$ are parallel to the ∇r, $\nabla \theta$, and $\nabla \phi$ directions, respectively.

Let $\mathbf{A} = \hat{\phi}A_\phi = \hat{\phi}\dfrac{\sin\theta}{r^2}$.

(a) Determine $\nabla \cdot \mathbf{A} = ?$

(b) Determine $\nabla \times \mathbf{A} = ?$

(c) Let $\partial \hat{r}/\partial\phi = c_1\hat{r} + c_2\hat{\theta} + c_3\hat{\phi}$. Find $[c_1, c_2, c_3] = ?$

7. (10 points)

Let $M = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 2 & 0 \\ -3 & 0 & 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, and $\mathbf{u} = M^{21}\mathbf{v}$.

Determine the column vector $\mathbf{u}$.

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