國立中央大學101學年度碩士班考試入學試題卷

所別：太空科學研究所碩士班 不分組(一般生)　科目：應用數學　共 2 頁 第 1 頁
太空科學研究所碩士班 不分組(在職生)

本科考試禁用計算器　　*請在試卷答案卷（卡）內作答

注意：作答時，如果只列出最後答案，卻沒有文字繪圖說明或計算步驟，該題將不予計分。

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1. (20 points) [(a) 5 points, (b) 5 points, (c) 5 points, (d) 5 points]

Consider the following function

$$f(x) = \frac{n}{\sqrt{2\pi}\sigma}\exp[-\frac{(x-\mu)^2}{2\sigma^2}]$$

where n, μ, and σ are positive real numbers.

(a) Find the location(s) where $\frac{d^2}{dx^2}f(x) = 0$.

(b) Sketch the function $f(x)$ from $x \to -\infty$ to $x \to +\infty$ and mark the location(s) where $\frac{d}{dx}f(x) = 0$ and the location(s) where $\frac{d^2}{dx^2}f(x) = 0$.

(c) Determine the definite integral $I_1 = \int_{-\infty}^{+\infty} f(x)dx$

(d) Determine the definite integral $I_2 = \int_{-\infty}^{+\infty} x^2 f(x)dx$

2. (20 points) [(a) 10 points, (b) 5 points, (c) 5 points]

Consider a particle with mass m, position $x(t)$, and x-component velocity $v_x(t)$, so that

$$\frac{dx(t)}{dt} = v_x(t)$$

If the particle is decelerated by a velocity-dependent force, such that

$$m\frac{dv_x(t)}{dt} = -bv_x(t)$$

where b is a positive real number. For $v_x(t)$ much less than the speed of light, the kinetic energy of the particle can be defined as $E_k = (1/2)mv_x^2$.

(a) Find the solutions of the functions $x(t)$ and $v_x(t)$ with initial conditions $v_x(t=0) = v_0$ and $x(t=0) = x_0$.

(b) Determine how the particle kinetic energy changes with particle's position x.

(c) Sketch the kinetic energy $E_k(x)$ as a function of particle's position x and determine where the particle will lose most of its kinetic energy.

注意：背面有試題

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3. (15 points) [(a) 5 points, (b) 5 points, (c) 5 points]

Let $M = \begin{bmatrix} a & 0 & -i\omega \\ 0 & b & 0 \\ +i\omega & 0 & c \end{bmatrix}$ where $i = \sqrt{-1}$

(a) Find M^{-1} (the inverse matrix of M)

(b) Find the eigen values of the matrix M.

(c) Show that the eigen vectors of the different eigen values of the matrix M are perpendicular to each other.

4. (20 points) [(a) 5 points, (b) 5 points, (c) 10 points]

Evaluate the following definite integrals, where $z_0 = 5 - 3i$ and $i = \sqrt{-1}$

(a) $I_1 = \int_{-\infty-i\infty}^{+\infty-i\infty} \frac{e^{-iz}}{z - z_0} dz$

(b) $I_2 = \int_{-\infty}^{+\infty} \frac{e^{-iz}}{z - z_0} dz$

(c) $I_3 = \int_{-\infty}^{+\infty} dk \int_{-\infty}^{+\infty} f(\xi)\, e^{ik(x-\xi)}\, d\xi$

5. (25 points) [(a) 5 points, (b) 5 points, (c) 5 points, (d) 5 points, (e) 5 points]

Let us consider a spherical coordinate system (r, θ, ϕ), where r is the radial distance from the origin; θ is the polar angle between the position vector $\mathbf{r} = \hat{r} r$ and the z-axis; ϕ is the azimuthal angle of the position vector $\mathbf{r}$ with respect to the x-z plane. The unit vectors $\hat{r}, \hat{\theta}, \hat{\phi}$ are parallel to the ∇r, $\nabla \theta$, and $\nabla \phi$ directions, respectively. Please determine the following vector differentiations.

(a) Let $\partial\hat{\theta}/\partial\theta = a_1\hat{r} + a_2\hat{\theta} + a_3\hat{\phi}$. Find a_1, a_2, a_3

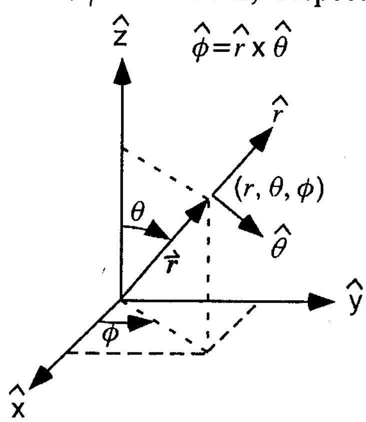
(b) Let $\partial\hat{r}/\partial\theta = b_1\hat{r} + b_2\hat{\theta} + b_3\hat{\phi}$. Find b_1, b_2, b_3

(c) Let $\partial\hat{r}/\partial\phi = c_1\hat{r} + c_2\hat{\theta} + c_3\hat{\phi}$. Find c_1, c_2, c_3

(d) Let $\partial\hat{\phi}/\partial\phi = d_1\hat{r} + d_2\hat{\theta} + d_3\hat{\phi}$. Find d_1, d_2, d_3

(e) Let $\partial\hat{\theta}/\partial\phi = e_1\hat{r} + e_2\hat{\theta} + e_3\hat{\phi}$. Find e_1, e_2, e_3

(以上每小題，三組係數同時答對才計分)



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