

國立中央大學99學年度碩士班考試入學試題卷

所別：地球物理研究所碩士班 不分組(一般生) 科目：微積分 共 2 頁 第 1 頁

本科考試禁用計算器

\*請在試卷答案卷(卡)內作答

作答時須列出完整計算過程

1. (a)  $\lim_{x \rightarrow 0} \frac{\cos 3x - 1}{5x^2} = ?$  (5%)

(b)  $\lim_{x \rightarrow 0} x \ln x = ?$  (5%)

2. (a) prove that  $\frac{d}{dx} \csc^{-1} x = \frac{-1}{x\sqrt{x^2-1}}$  (5%)

(b)  $y(x) = \left( \frac{x^{1/2} + 1}{x^{1/2} - 1} \right)^{1/2}$ ,  $\frac{dy}{dx} = ?$  (5%)

3. (a)  $\int_{-\infty}^{\infty} e^{-x^2} dx = ?$  (5%)

(b)  $\int e^{ax} (\cos bx) dx = ?$  (5%)

4. Solve  $y'' + 3y' + 2y = 30e^{2x}$  (10%)

5. What is Divergence Theorem of Gauss? Apply it to evaluate

$\iint_S (2x\hat{i} + 2y\hat{j} + 2z\hat{k}) \cdot \hat{n} dA$  over the sphere  $S: x^2 + y^2 + z^2 = 4$ . (10%)

6. Given three points (0, 0), (1, 1) and (2, 3), use least square method to determine the best fitting straight line. (10%)

7. Find the eigenvalues and eigenvectors of  $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$  (10%)

參考用

注意：背面有試題

8. 以泰勒級數將  $\frac{1}{(1+x^2)}$  在  $x_0=1$  處，展開至  $(x-x_0)^3$  項。(10%)

9. Find the Fourier series of  $f(x) = x + \pi$  ( $-\pi < x < \pi$ ) and  $f(x+2\pi) = f(x)$ .  
(10%)

10.  $e^{i(\omega t - \vec{k} \cdot \vec{x})}$  represents a propagating plane wave in 3-D space, where

$\vec{k} = k_x \hat{i} + k_y \hat{j} + k_z \hat{k}$  is the wave number vector indicating the direction of

propagation and  $\vec{x} = x\hat{i} + y\hat{j} + z\hat{k}$  is the position vector.

(a) Given a scalar potential  $\phi(\vec{x}, t) = e^{i(\omega t - \vec{k} \cdot \vec{x})}$ , show that the displacement of its gradient is parallel to the direction of propagation. (5%)

(b) Given a vector potential  $\vec{\gamma}(\vec{x}, t) = (A_x, A_y, A_z) e^{i(\omega t - \vec{k} \cdot \vec{x})}$ , show that the displacement of its curl is perpendicular to the direction of propagation. (5%)

參考用

注意：背面有試題