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所別：地球物理研究所碩士班　　科目：應用數學

1. Solve the following ordinary differential equations for $y = y(x)$:

(a) [10%] $e^{x}\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2(x+1)y^{2}, \quad y(0) = \dfrac{1}{6}$

(c) [20%] $\dfrac{\mathrm{d}^{2}y}{\mathrm{d}x^{2}} - 4y = e^{-2x} - 2x, \quad y(0) = 0, \quad \dfrac{\mathrm{d}y}{\mathrm{d}x}(0) = 0$

2. For the given matrices: $\mathbf{A} = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 2 & -3 \\ 0 & 2 \\ 0 & 1 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$, evaluate:

(a) [10%] $\mathbf{A}^{\mathrm{T}} \cdot \mathbf{B}$　　(b) [10%] $\mathbf{C}^{-1}$

3. For any twice continuously differentiable scalar function f and vector function $\vec{u}$, verify the following identities:

(a) [10%] $\nabla \times (\nabla f) = 0$　　(b) [10%] $\nabla \cdot (\nabla \times \vec{u}) = 0$

4. [30%] Consider a one-dimensional heat equation along the x-axis: $\dfrac{\partial u}{\partial t} = \dfrac{\partial^{2} u}{\partial x^{2}}$, where $u = u(x,t)$ and $0 < x < \pi$. Solve the heat equation subject to the boundary conditions: $\dfrac{\partial u}{\partial x}(0,t) = \dfrac{\partial u}{\partial x}(\pi,t) = 0$ and the initial condition: $u(x,0) = \begin{cases} 0, & 0 < x < \pi/2 \\ 1, & \pi/2 < x < \pi \end{cases}$.