

系所別: 地球物理研究所
應用地質研究所

科目:

微積分

參考用

1. (a) $\lim_{x \rightarrow 4} \frac{x^2 - x - 12}{x^2 - 3x - 4} = ?$. (8%)

(b) if $\lim_{x \rightarrow 2} \frac{ax^2 + 2x + b}{x - 2} = 3$, find a, b . (8%)

2. Find the 4th degree Taylor formula of $\cos x$ at $\pi/3$. (12%)

3. Consider a circuit with $L = 3$ henrys, $R = 9$ ohms and an alternating current generator that supplies a voltage of $E(t) = 12 \sin(9t)$ volts (Figure 1). If $I = 0$ at $t = 0$ (when the switch S is closed, find I at time t and note its behavior for large t . (12%)

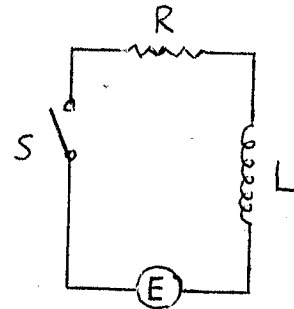


Figure 1.

4. Use Stoke's theorem to evaluate $\oint_C \vec{F} \cdot \vec{T} ds$, where

$\vec{F} = (2z)\vec{i} + (8x - 3y)\vec{j} + (3x + y)\vec{k}$ and C is the triangular curve of Figure 2. (12%)

5. Find the center of mass of a lamina in the shape of a quarter-circle of radius a whose density is proportional to the distance from the center of the circle. (Figure 3). (12%)

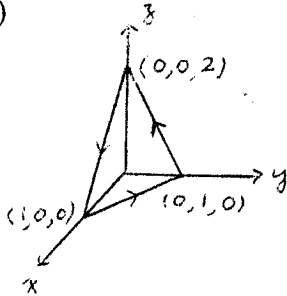


Figure 2.

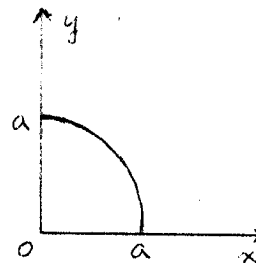


Figure 3.

6. As a solid right circular cylinder is heated, its radius r and height h increase; hence, so does its surface area S . Suppose that at the instance when $r = 10$ centimeters and $h = 100$ centimeters, r is increasing at 0.1 centimeters per hour and h is increasing at 0.3 centimeters per hour. How fast is S increasing at this instant? (12%)

7. Find $\int (\sec^3 \theta) d\theta$. (12%)

8. Carbon 14, one of the three isotopes of Carbon, is radioactive and decays at a rate proportional to the amount present. Its half-life is 5730 years, that is, it takes 5730 years for a given amount of carbon-14 to decay to one-half its original size. If 15 grams were present originally, how much will be left after 3000 years. (12%)