國立中央大學111學年度碩士班考試入學試題

所別： 地球科學學系地球物理 碩士班 不分組(一般生)　　共2頁 第1頁
地球科學學系地球物理 碩士班 不分組(在職生)

科目： 微積分

作答時須列出完整計算過程

1. (a) (5%) $\lim_{x\to 0}\frac{e^{2x}-e^{x}}{x}=?$

   (b) (5%) $\lim_{\theta\to 0}\frac{sin\theta}{\theta+tan\theta}=?$

2. (a) (5%) $\int x^3 cosx^2 dx=?$

   (b) (5%) $\int_{-\infty}^{\infty} x^2 dx=?$

3. (a) (5%) Apply Rodrigues's formula $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}[(x^2-1)^n]$ to find Legendre functions $P_0(x)$, $P_1(x), P_2(x), P_3(x)$.

   (b) (5%) Given $P_4(x)=\frac{1}{8}(35x^4-30x^2+3)$, derive the Fourier-Legendre series for $f(x)=7x^4+10x^3+6x^2+3x+5$, namely, $f(x)=\sum_{m=0}^{\infty} a_m P_m(x)$.

4. (10%) Solve the initial value problem
   $$y''+y'+0.25y=0,\quad y(0)=3.0,\quad y'(0)=-3.5$$

5. (10%) Show that if $re^{i\theta}=\frac{a-ib}{a+ib}$, then $r=1$, $\theta=-2tan^{-1}\frac{b}{a}$

6. (10%) Use the method of separating variables to solve the one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$, for the vibrations of an elastic string of length L.
   The boundary conditions are $u(0,t)=0$, $u(L,t)=0$ for all t.
   The initial conditions are $u(x,0)=f(x)$, $u_t(x,t)|_{t=0}=0$.

注意:背面有試題

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7. (a) (4%) What is Heaviside function $u(t-a)$ and find its Laplace Transform $\mathcal{L}\{u(t-a)\}$?
   (b) (6%) Find the inverse transform *f(t)* of

$$F(s) = \frac{e^{-s}}{s^2+\pi^2} + \frac{e^{-3s}}{(s+2)^2}$$

8. (10%) Find the **odd** periodic expansions of the function (half-range expansion)

$$f(x) = \begin{cases} \frac{2k}{L}x & \text{if} \quad 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x) & \text{if} \quad \frac{L}{2} < x < L. \end{cases}$$

9. (10%) Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

10. (a) (4%) Explain thermal conductivity and specific heat.
    (b) (2%) What is divergence theorem?
    (c) (4%) Model heat flow from a body in space to derive heat equation.

注意:背面有試題