

所別：地球科學學系地球物理碩士班 不分組(一般生) 科目：微積分 共 2 頁 第 1 頁  
地球科學學系地球物理碩士班 不分組(在職生)

本科考試禁用計算器

\*請在試卷答案卷(卡)內作答

參考用

作答時須列出完整計算過程(共兩頁)

$$1. (a) \lim_{x \rightarrow 5} \frac{\frac{x}{x+2} - \frac{5}{7}}{x-5} = ? \quad [5\%]$$

$$(b) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin(\cos x)}{\cos x} = ? \quad [5\%]$$

$$2. (a) f(x) = \tan^{-1} x + \frac{1}{3} \tan^{-1}(x^3), \quad \frac{df(x)}{dx} = ? \quad [5\%]$$

$$(b) y = a^x, \quad \frac{dy}{dx} = ? \quad [5\%]$$

$$3. (a) \int_{-\infty}^{\infty} e^{-x^2} dx = ? \quad [5\%]$$

$$(b) \int_2^3 \frac{(x+1)dx}{\sqrt{x^2+2x+3}} = ? \quad [5\%]$$

$$4. \text{ Solve the initial value problem } \frac{d^2 y}{dx^2} + y = 2x^2, \quad y(0) = 0, \quad y'(0) = 4. \quad [10\%]$$

5. Find the eigenvalues and normalized eigenvectors of the matrix. [10%]

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & -3 \\ 3 & -3 & -3 \end{pmatrix}$$

注意：背面有試題

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6. Find a solution formula of  $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$  satisfying the initial condition

$u(x, 0) = f(x)$  and the boundary condition  $\frac{\partial u(0, t)}{\partial x} = 0, \frac{\partial u(L, t)}{\partial x} = 0$ . [10%]

7. Find the odd periodic extension of the function (half-range expansion) [10%]

$$f(x) = \begin{cases} \frac{2k}{L}x & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x) & \text{if } \frac{L}{2} < x < L. \end{cases}$$

8. Find an integrating factor and solve the initial value problem

$$(e^{x+y} + ye^y)dx + (xe^y - 1)dy = 0, \quad y(0) = -1. \quad [10\%]$$

9. Compute the area of the region  $S$  between the graphs of  $f$  and  $g$  over the interval  $[-1, 2]$  if  $f(x) = x$  and  $g(x) = x^3/4$ . [10%]

10. 求 $(-2, -2), (0, 1), (1, 2), (2, 4)$ 所決定的最佳直線  $y = ax + b$ , 使得其誤差平方和最小(the least squares solution) [10%].

注意：背面有試題